Longitudinal observations

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LEAD symposium, EDEG 2014
31 March 2014

http://BendixCarstensen.com/SDC/LEAD

Two observation points

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31 March 2014
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(`twopoints`)

Basic set-up: Two time points

Measurements at two time points

- Randomized study:

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- Randomized study:
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  - Describe population processes
  - Nothing special about any one point of observation
  - — except that this was the first measuring occasion.

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- Thus we know:
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  - ny difference at follow-up due to intervention.

Simple approaches

- Compute the change in each group

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- Not quite so: Regression to the mean

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- — hence with a random part which on average is smaller.

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B_i	μ	σ
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So x large (*i.e.* $x > \mu$) means that the conditional mean is **smaller** than Δ - the **true** difference.

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$$\rho = \text{corr}(F, B) = \text{corr}(y_{t2}, y_{t1}) = \frac{\tau^2}{\tau^2 + \sigma^2}$$

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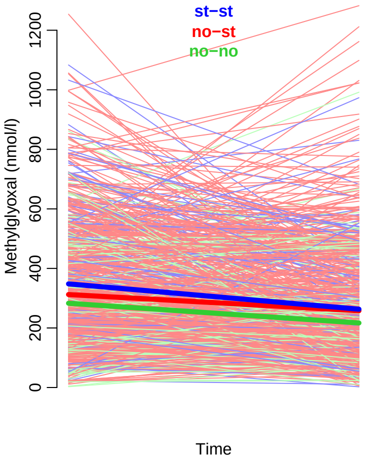

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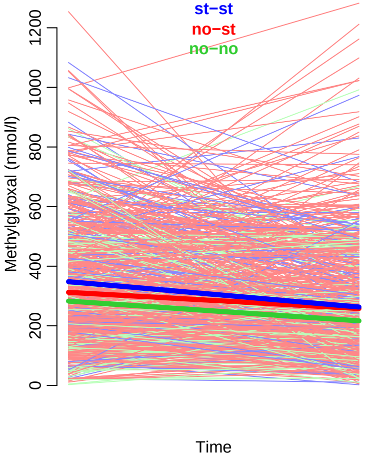

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Variation between line-midpoints

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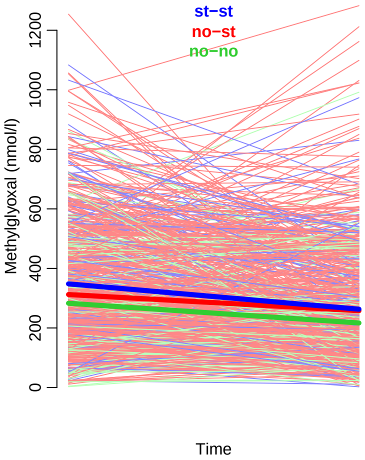

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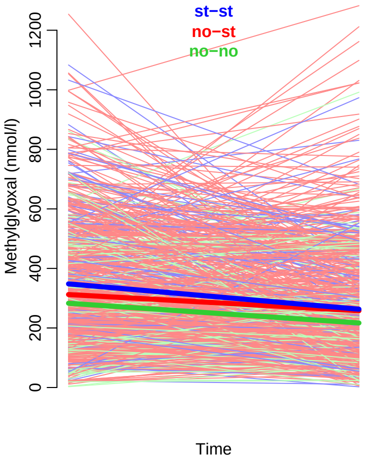

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σ is the variation round these slopes

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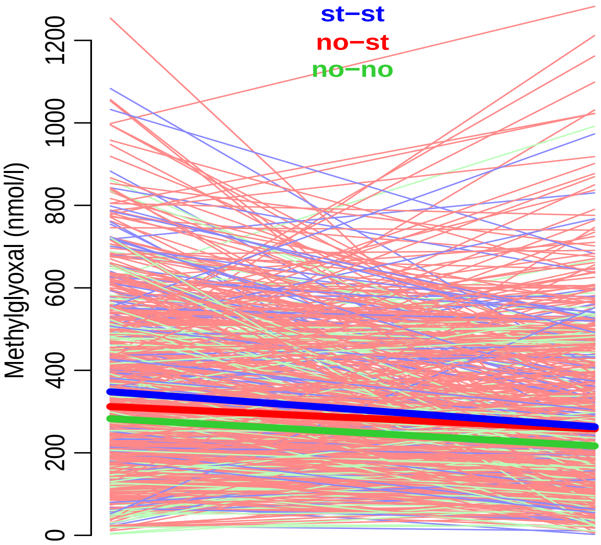

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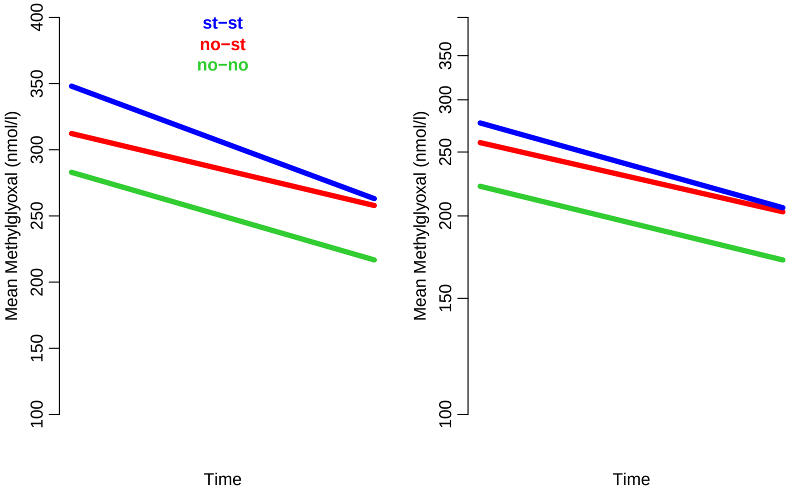

Source: Troels Mygind Jensen & Addition-PRO

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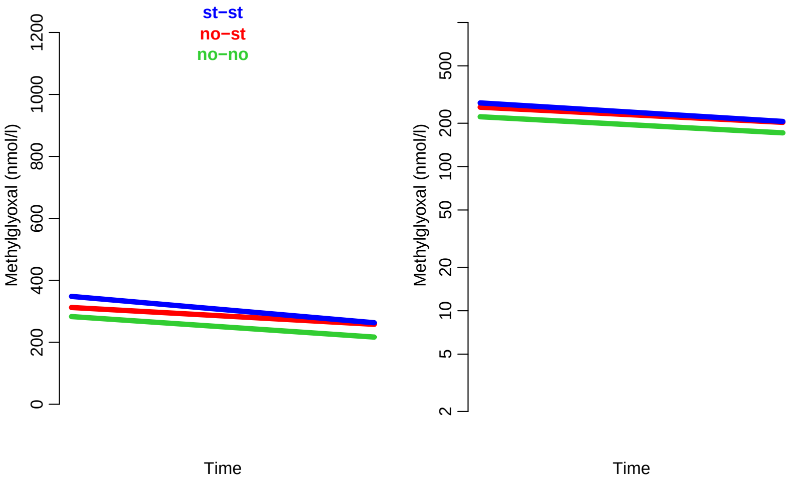

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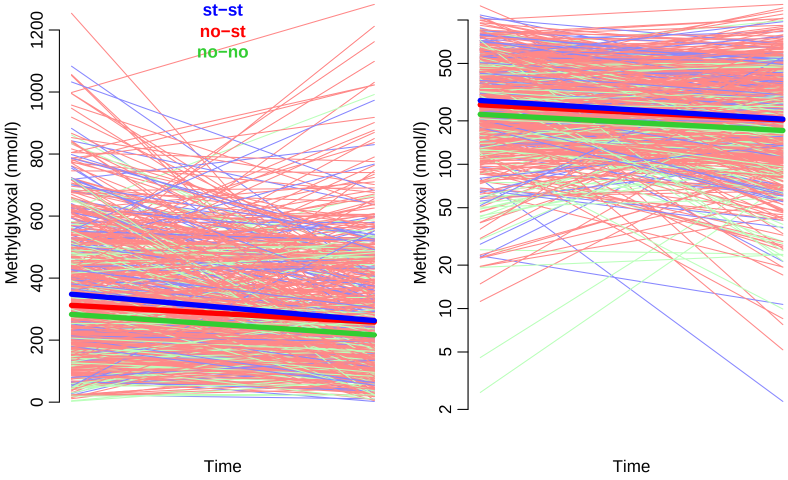

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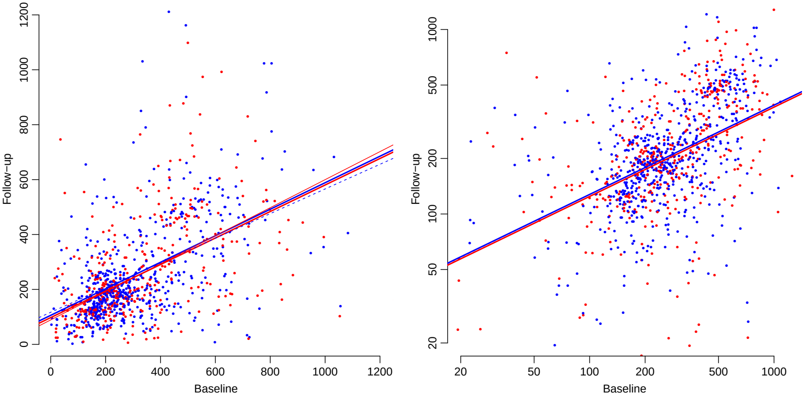

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Analysis by lm I

```
cf <- coef( m0 <- lm( log10(mf) ~ log10(mb) + factor(gr), data=
round( ci.lin( m0 ), 2 )
```

```
              Estimate StdErr      z    P  2.5% 97.5%
(Intercept)       1.14   0.07 15.50 0.00  0.99  1.28
log10(mb)         0.48   0.03 16.26 0.00  0.43  0.54
factor(gr)1      -0.01   0.02 -0.59 0.56 -0.05  0.03
```

Multiple measurements

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(`multpt`)

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- Model data by random effects models for **mean** and **between person variation**
- Limited amount of information per person.

Random effects — error structure

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- Random slopes, random higher-order terms can be added
- Neither approach requires the same number of timepoints (let alone identical timepoints) between persons' measurements.
- This is how the world usually looks.

Data structure: "long" format

- Always advisable to have data in the long form:

```
head( gluc )
     id  fpg    ds       time gruppe   end   tfe
1  4521 5.35 13895 -10.512011      0 17724 -3829
2  4521 5.30 15890  -5.035003      0 17724 -1834
3  4521 5.90 17724   0.000000      0 17724     0
4 10613 5.00 12116   0.000000      0 12116     0
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2  4521 5.30 15890  -5.035003      0 17724 -1834
3  4521 5.90 17724   0.000000      0 17724     0
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Simple model for repeated measures

Measurement on individual i at timepoint t

$$y_{ti} = \mu + [\mathsf{cov}] + a_i + e_{it}$$

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e_{it} is a random effect representing the measurement error on any measurement

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Standard deviation of the a_is is τ, say;
you get an estimate of this from statistics programmes.

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- This is the way to report between person variation [**?**]

Extended model: Random slopes

Measurement on individual i at timepoint t

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The variation in $a_i + b_i t$ is now the **between** person variation; depending on t.

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Changing the times individually

Bendix Carstensen

LEAD
31 March 2014
LEAD symposium, EDEG 2014
`http://BendixCarstensen.com/SDC/LEAD`

(`reshuf`)

Relative changes of times

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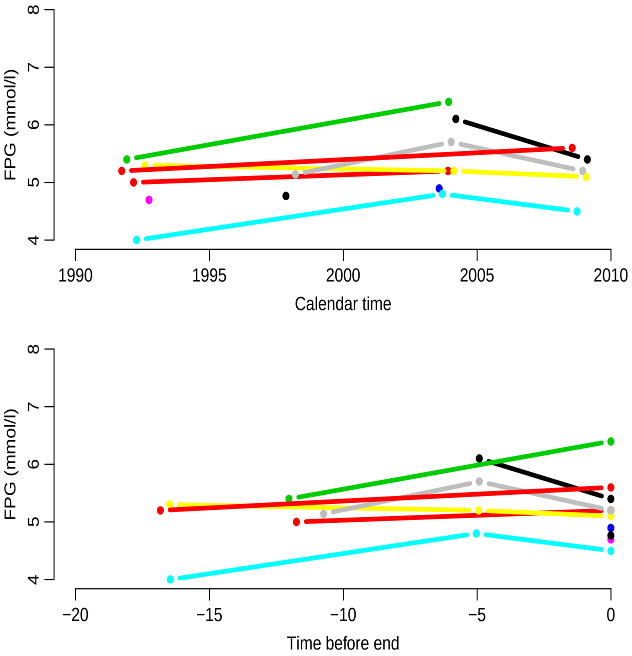
- Change of the statistical model in terms of interpretation


FPG (mmol/l)
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Calendar time
FPG (mmol/l)
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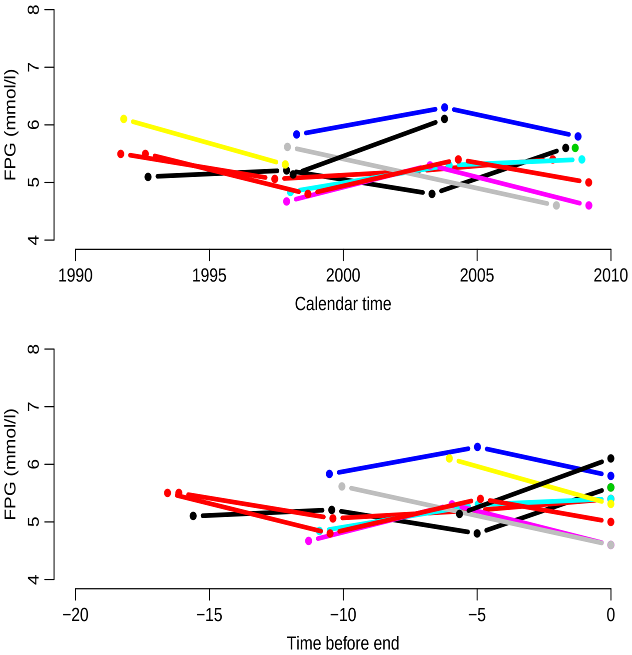
Time before end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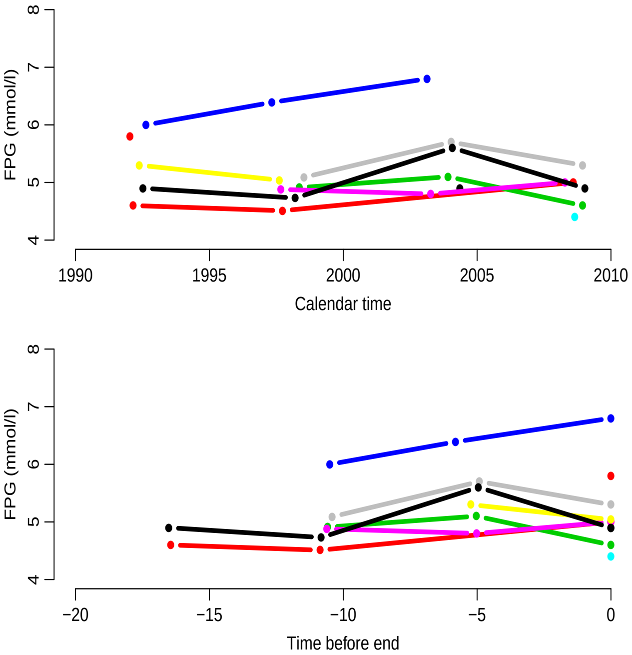
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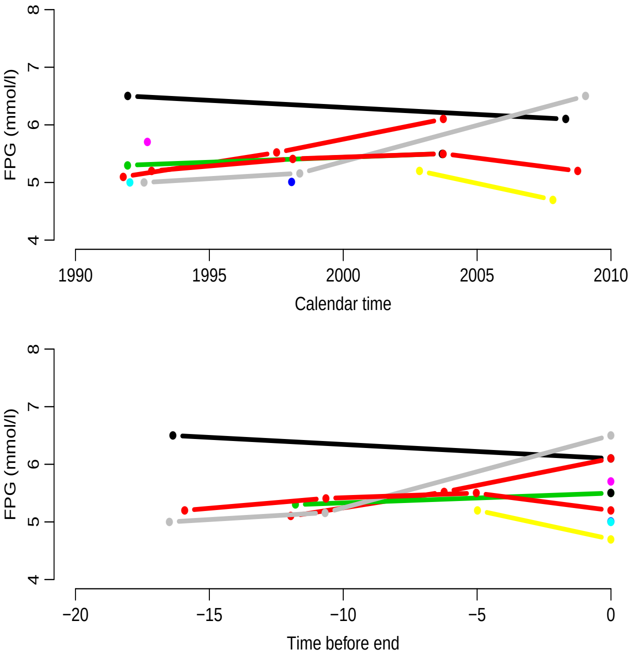
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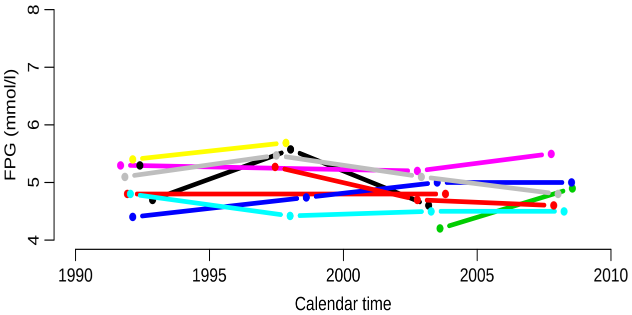
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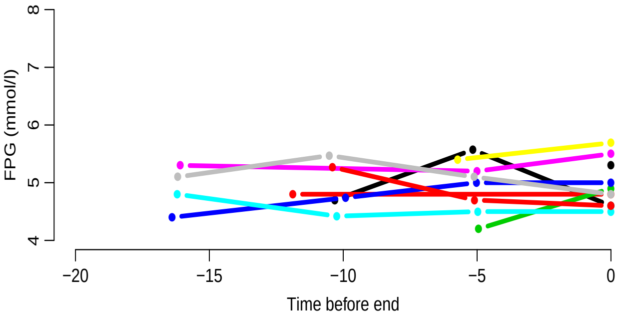
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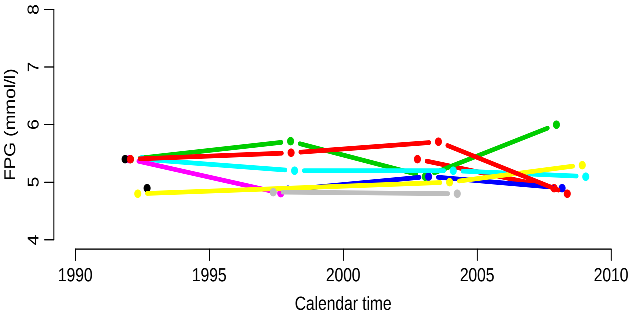
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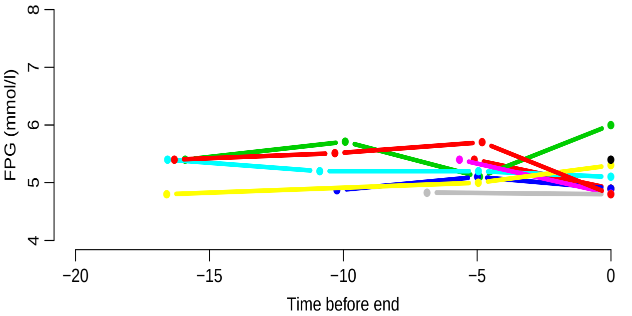
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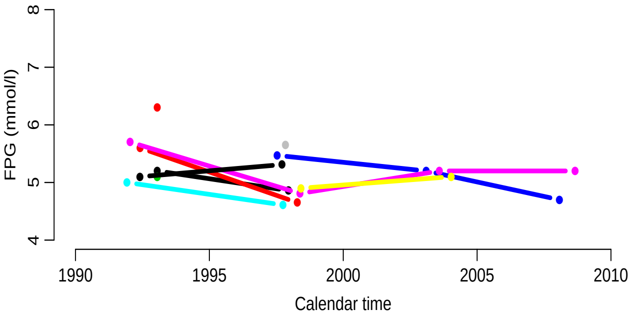
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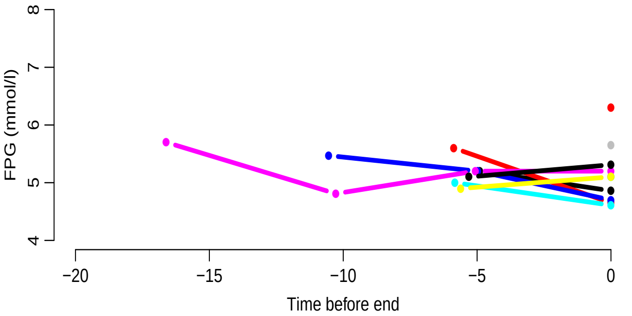
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Not meaningful for covariates:

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- ... some may even think it is the unconditional.

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- Is time just a surrogate for age???

Conclusions

Bendix Carstensen

LEAD
31 March 2014
LEAD symposium, EDEG 2014
`http://BendixCarstensen.com/SDC/LEAD`

(`concl`)

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- — the variation between and within persons are also important

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- — that captures what you want to know about.

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  - — possibly varying along the time scale,