

Adjust for baseline — or not in studies of change

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Basic set-up

Measurement at two time points

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- ▶ Randomized study:

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 - ▶ in **observational** studies covariate effects at baseline may be of interest

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- ▶ ... regression to the mean

Example from Vickers et al.[?]

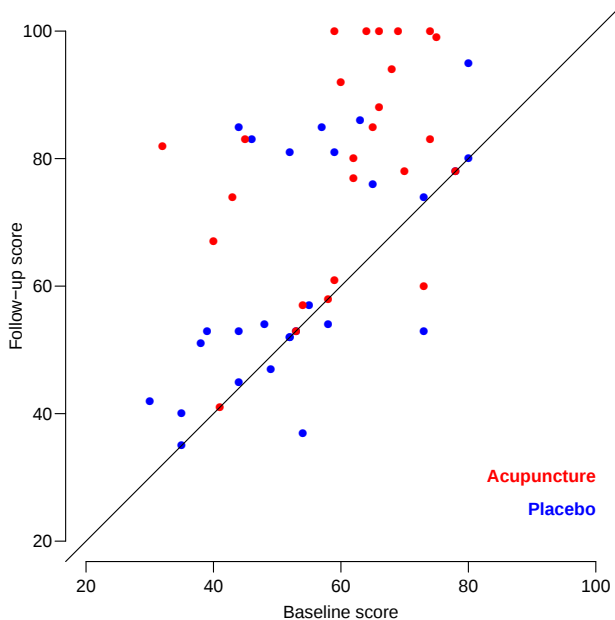
```
> library( Epi )
> library( foreign )
> acp <- read.dta( "../data/sportsmen.dta" )[, -4]
> names( acp ) <- c("bl", "fu", "gr")
> acp$gr <- factor( acp$gr, labels=c("Placebo", "Acupuncture") )
> str( acp )

'data.frame': 54 obs. of 3 variables:
 $ bl: num  59 53 46 38 52 63 30 73 44 48 ...
 $ fu: num  81 53 83 51 81 86 42 74 45 54 ...
 $ gr: Factor w/ 2 levels "Placebo", "Acupuncture": 1 1 1 1 1 1 1 1 1 1 ...
> head( acp )
   bl fu   gr
1  59 81 Placebo
2  53 53 Placebo
3  46 83 Placebo
4  38 51 Placebo
5  52 81 Placebo
6  63 86 Placebo
```

Randomized to
acupuncture /
placebo

Outcome:
Shoulder pain rating
(scale from 0 to 100)

Change is the vertical
distance from the
identity line to the point

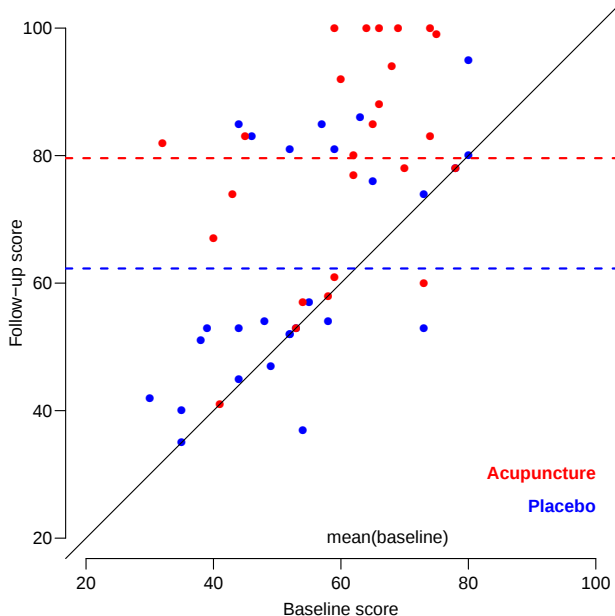


Follow-up analysis

$$y_{fi} \mid \mu_g$$

Randomized study:
Analysis of the follow-up
measurements is in
principle unbiased

because the
baseline-distribution is
the same in the two
groups.



Analysis of follow-up values

```
> # Follow-up
> fm <- with( acp, tapply( fu, gr, mean ) )
> c( fm, diff(fm) )
```

	Placebo	Acupuncture	Acupuncture
	62.2963	79.6000	17.3037

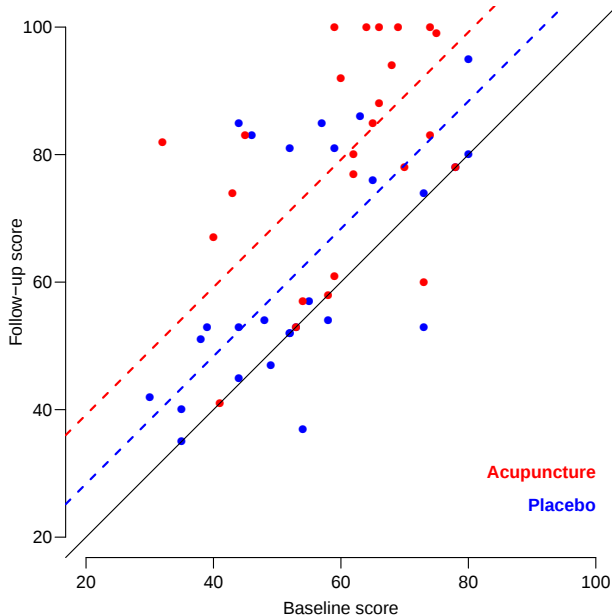
```
> mf <- lm( fu ~ gr, data = acp )
> round( ci.lin( mf ), 4 )
```

	Estimate	StdErr	z	P	2.5%	97.5%
(Intercept)	62.2963	3.3783	18.4400	0e+00	55.6749	68.9177
grAcupuncture	17.3037	4.8723	3.5515	4e-04	7.7542	26.8532

Analysis of change-scores

$$y_{fi} - y_{bi} - \mu_g$$

The change score result (treatment effect) is the vertical difference between the lines.



Analysis of change-scores

```
> # Follow-up
> cm <- with( acp, tapply( fu-bl, gr, mean ) )
> c( cm, diff(cm) )
```

	Placebo	Acupuncture	Acupuncture
	8.37037	19.20000	10.82963

```
> mc <- lm( fu-bl ~ gr, data = acp )
> round( ci.lin( mc ), 4 )
```

	Estimate	StdErr	z	P	2.5%	97.5%
(Intercept)	8.3704	2.9480	2.8394	0.0045	2.5924	14.1483
grAcupuncture	10.8296	4.2516	2.5472	0.0109	2.4966	19.1627

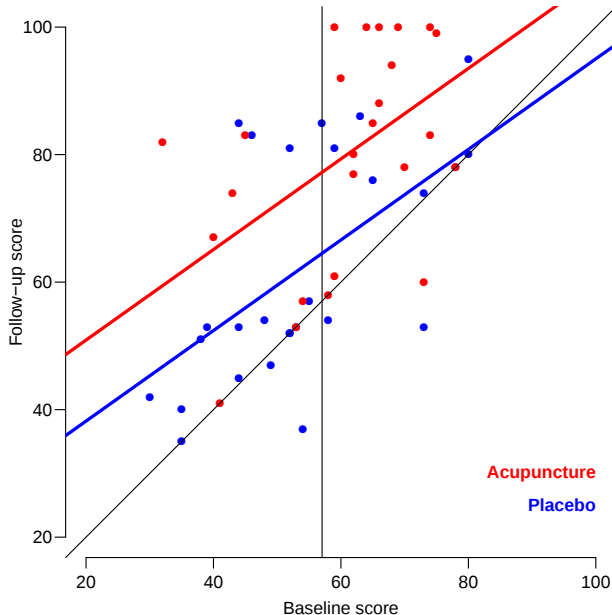
Conditioning on baseline

$$y_{1i}|y_{0i} \mu_g$$

Accounts for possible imbalances in baseline distribution

Controlling for confounding by baseline value

Effect is vertical distance between lines



Conditioning on baseline

```
> ml <- lm( fu ~ bl + gr, data = acp )  
> round( ci.lin( ml ), 4 )
```

	Estimate	StdErr	z	P	2.5%	97.5%
(Intercept)	23.9973	9.1092	2.6344	0.0084	6.1435	41.8511
bl	0.7102	0.1602	4.4323	0.0000	0.3962	1.0243
grAcupuncture	12.7057	4.2857	2.9647	0.0030	4.3059	21.1056

- ▶ $y_{1i} = M + By_{0i} + D_g$
- ▶ Treatment effect is 12.7 points:
 - ▶ change on placebo:
$$M + (B - 1)y_{0i} + D_{pl} = 23.997 + 0.290y_{01} + 0$$
 - ▶ change on treatment:
$$M + (B - 1)y_{0i} + D_{tr} = 23.997 + 0.290y_{01} + 12.706$$

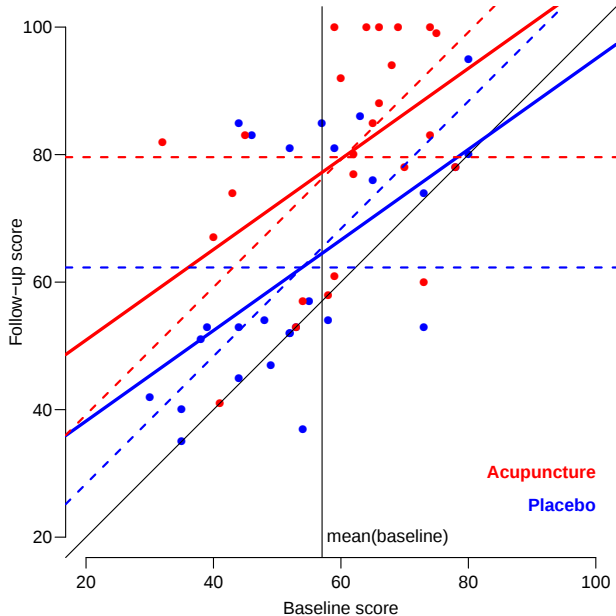
Conditioning on baseline

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 - ▶ change on treatment:
$$M + (B - 1)y_{0i} + D_{tr} = 23.997 + 0.290y_{01} + 12.706$$
- ▶ Change from baseline depends on baseline value
- ▶ **Difference** in change between does **not**
- ▶ ...but that is a model **assumption**.

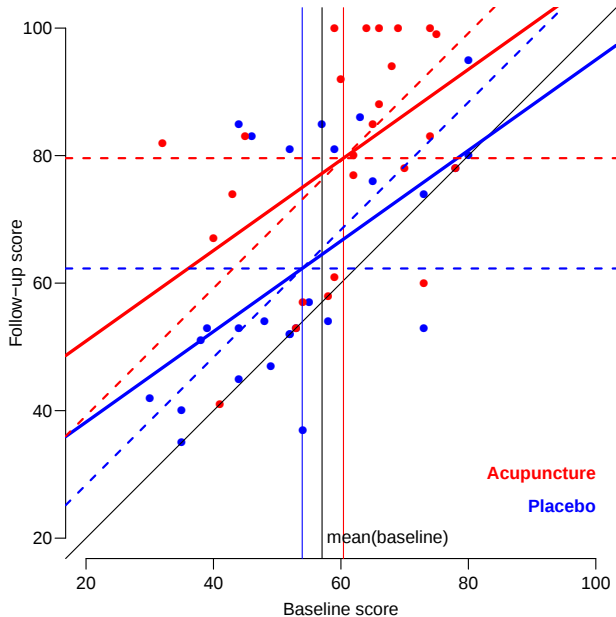
Comparing the three approaches

Effect is vertical distance
between lines

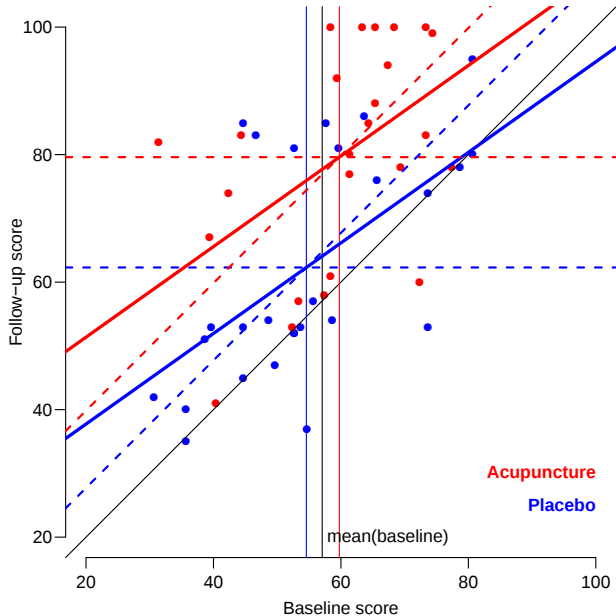
Three sets of lines
— three different
estimates.



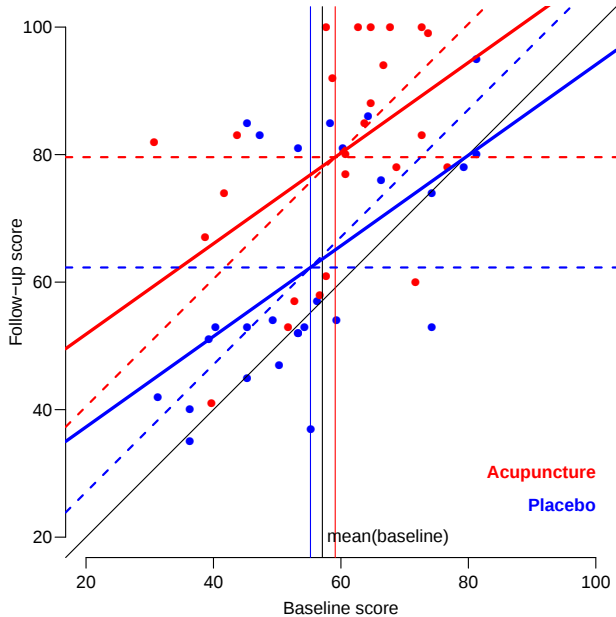
It all depends on the
baseline imbalance



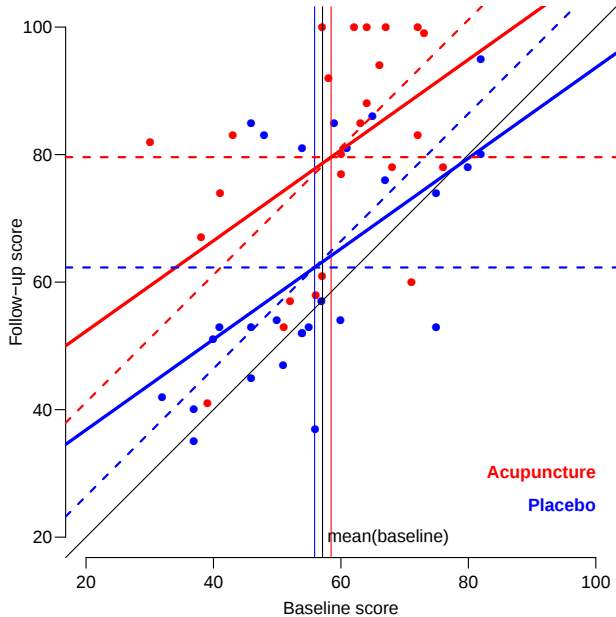
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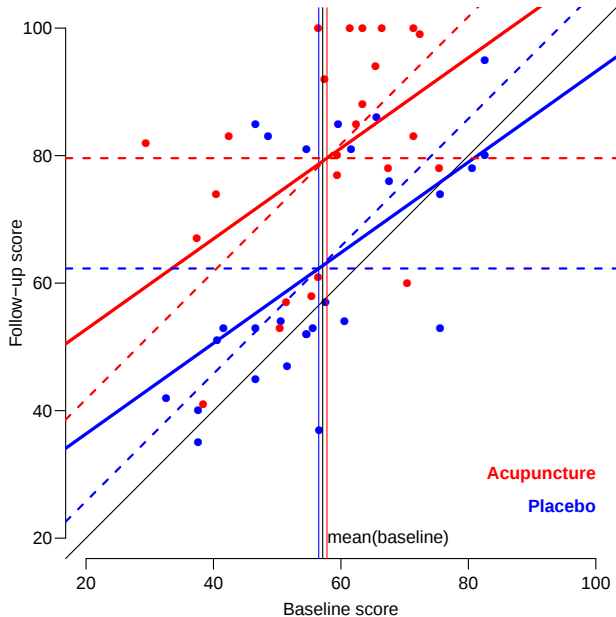
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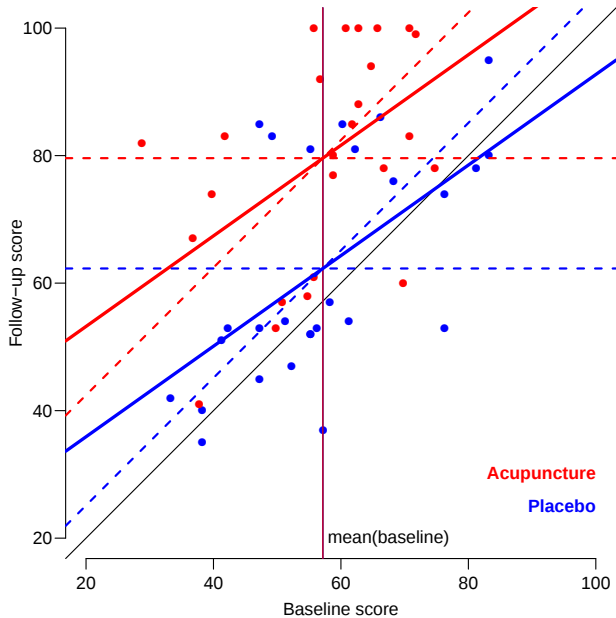
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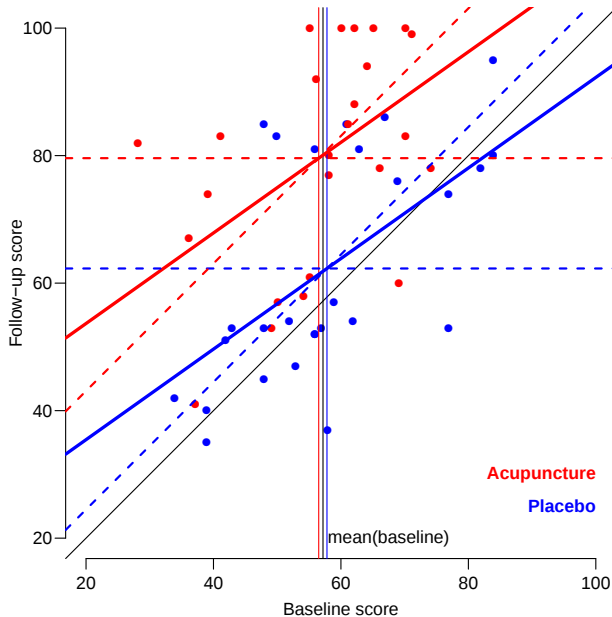
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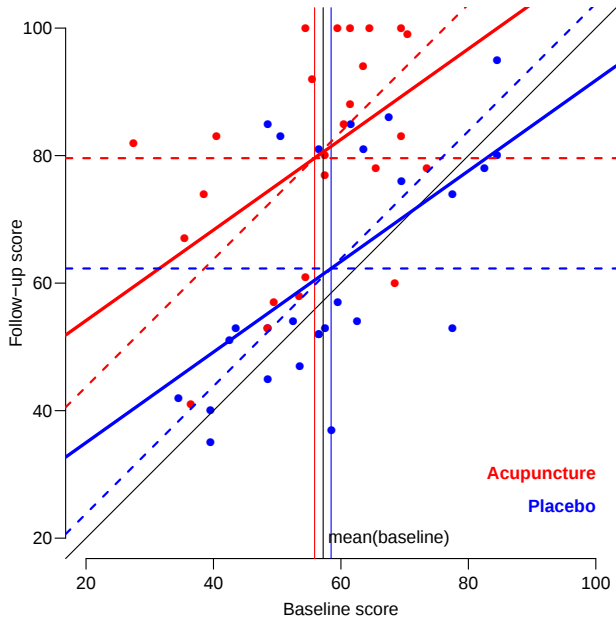
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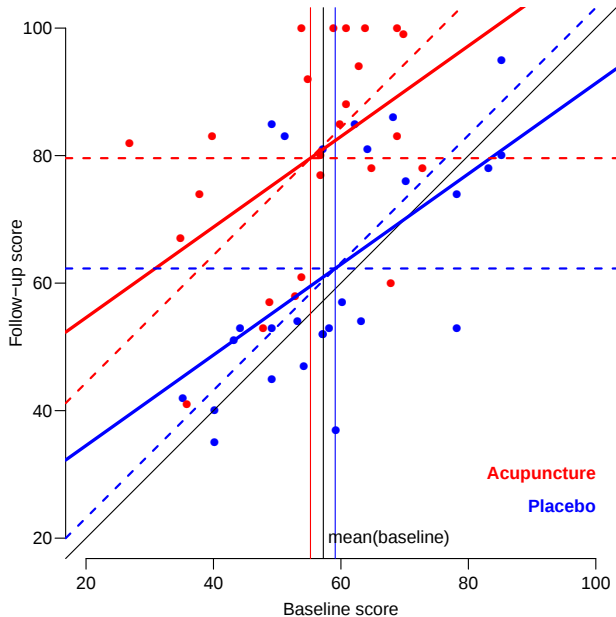
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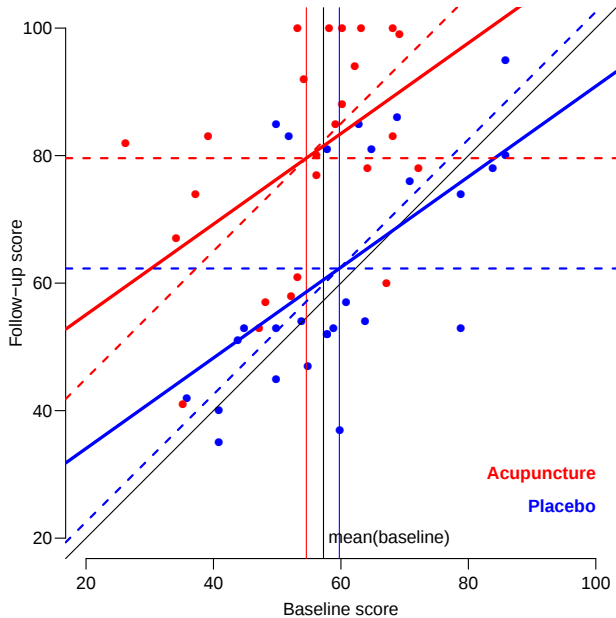
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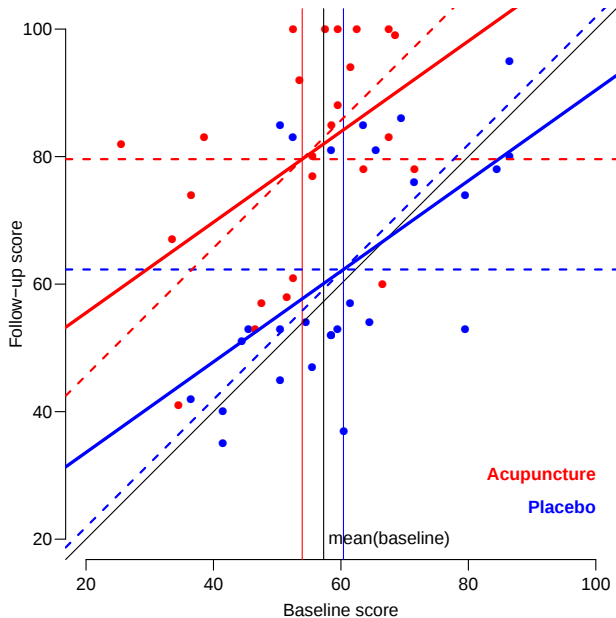
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Random effects model

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- ▶ Greater flexibility:
 - ▶ accommodate more than two measurements
 - ▶ not necessarily the same no. measurements per person
 - ▶ accommodate **actual** measurement times
- ▶ For two points it is close to the ANCOVA approach, but not the same

Random effects model

Random effects model: $\Rightarrow$ data in the long format:

```
> lg <- reshape( acp, varying=1:2, v.names="score", direction="long" )
> head( lg )
```

	gr	time	score	id
1.1	Placebo	1	59	1
2.1	Placebo	1	53	2
3.1	Placebo	1	46	3
4.1	Placebo	1	38	4
5.1	Placebo	1	52	5
6.1	Placebo	1	63	6

```
> str( lg )
'data.frame': 108 obs. of 4 variables:
 $ gr      : Factor w/ 2 levels "Placebo","Acupuncture": 1 1 1 1 1 1 1 1 1 1 1 ...
 $ time    : int  1 1 1 1 1 1 1 1 1 1 1 ...
 $ score   : num  59 53 46 38 52 63 30 73 44 48 ...
 $ id      : int  1 2 3 4 5 6 7 8 9 10 ...
- attr(*, "reshapeLong")=List of 4
 ..$ varying:List of 1
 .. ..$ score: chr  "bl" "fu"
 .. ..- attr(*, "v.names")= chr  "score"
 .. ..- attr(*, "times")= int  1 2
```

Random effects model

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> library( lme4 )  
> mr <- lmer( score ~ gr + gr:factor(time) + (1|id), data=lg )  
> round( ci.lin( mr ), 2 )
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	Estimate	StdErr	z	P	2.5%	97.5%
(Intercept)	53.93	2.99	18.03	0.00	48.06	59.79
grAcupuncture	6.47	4.31	1.50	0.13	-1.98	14.93
grPlacebo:factor(time)2	8.37	2.95	2.84	0.00	2.59	14.15
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- ▶ baseline mean in Placebo is 53.93

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Random effects model

Formally the model is:

$$\begin{aligned}y_{it} &= \mu + \delta_g + \beta_t + \gamma_{gt} + \eta + a_i + e_{it} \\i &= 1, \dots, I, \quad t = 0, 1, \quad g = \text{pl, int} \\a_i &\sim \mathcal{N}(0, \tau^2), \quad e_{it} \sim \mathcal{N}(0, \sigma^2)\end{aligned}$$

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... this is a 2-dimensional normal distribution, and in this

$$y_1|y_0 \sim \mathcal{N}\left(\mu_1 + \frac{\rho\sigma_1}{\sigma_0}(y_0 - \mu_0), \sigma_1^2(1 - \rho^2)\right)$$

... don't worry

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follow-up measurements given baseline:

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The random effects model shows how to compute the **conditional** distribution (well, mean) of:

follow-up measurements given baseline:

$$E(y_1|y_0) = \mu_1 + \frac{\rho\sigma_1}{\sigma_0}(y_0 - \mu_0),$$

μ_1, μ_0 are follow-up and baseline means,

σ_1, σ_0 are baseline variances, ρ the correlation

— all functions of the parameters specified in the random effects model.

Conditional mean

```
> mR <- lmer( score ~ gr*factor(time) + (1|id), data=lg )
> round( ci.lin( mR ), 2 )
```

	Estimate	StdErr	z	P	2.5%	97.5%
(Intercept)	53.93	2.99	18.03	0.00	48.06	59.79
grAcupuncture	6.47	4.31	1.50	0.13	-1.98	14.93
factor(time)2	8.37	2.95	2.84	0.00	2.59	14.15
grAcupuncture:factor(time)2	10.83	4.25	2.55	0.01	2.50	19.16

```
> cf      <- fixef( mR )                # regression coef
> tausq   <- as.numeric( VarCorr( mR )$id ) # tau-squared
> sigsq   <- attr( VarCorr( mR ), "sc" )^2  # sigma-squared
> rho     <- tausq/(tausq+sigsq)           # rho - correlation
```

Hence what we need to compute is:

```
> ( 1- rho ) * cf[2] + cf[4]
grAcupuncture
13.97486
```

— close to the intervention effect 12.7 in the conditional model.

Treatment effects from different models

Treatment effect from model:	Estimate	s.e.	Cond.diff
Conditional (ANCOVA)	12.71	4.29	12.71
Random effects:			
identical baseline	13.94	3.72	13.94
different baseline	10.83	4.25	13.97
Change score difference	10.83	4.25	
Follow-up difference	17.30	4.87	

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- ▶ Omitting it does not, and gives the conditional difference as an explicit parameter.

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- ▶ Since your data are not getting smaller and simpler:
- ▶ ...you might as well get used to it sooner than later.