The resurrection of time as a continuous concept in biostatistics, demography and epidemiology

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Inference in Multistate models

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1. Do not condition on the future
2. Do not regard individuals at risk after they have died
3. Stick to this world

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- Including persons' follow-up in the wrong state
- …namely one reached some time in the future
- Normally caused by classification of **persons** instead of classification of **follow-up time**

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  - other covariates

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- Covariates relate to each interval of follow-up
- Allows **multiple** timescales, *e.g.* age, duration, calendar time

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…which is indeed **not** of this world.

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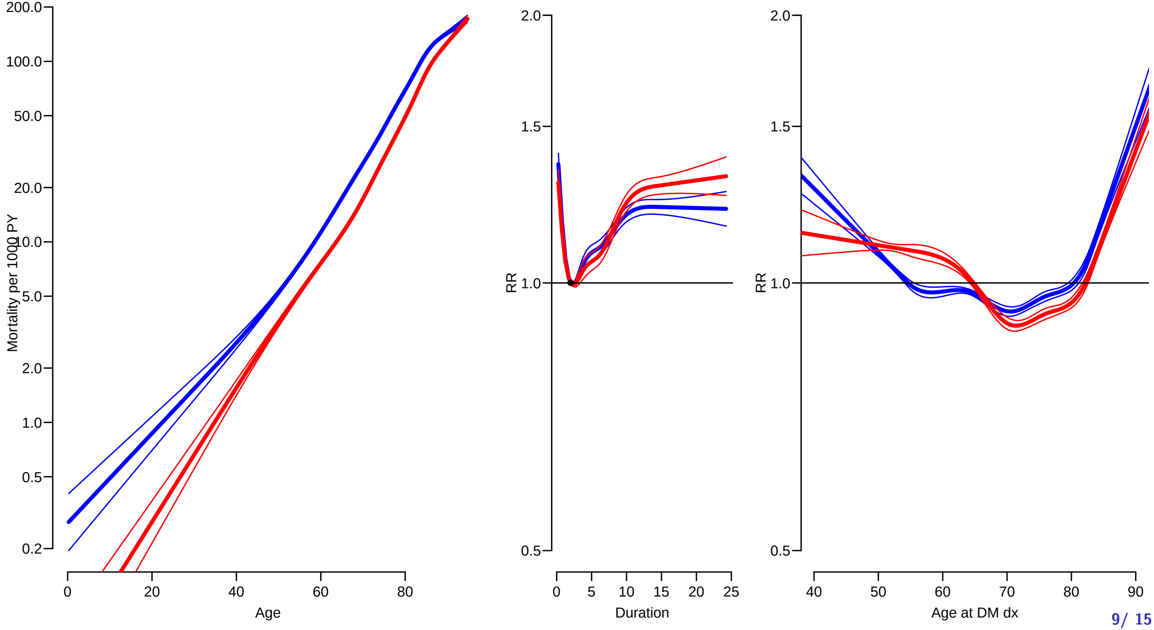
- — this is the APC-modeling problem again


200.0
100.0
50.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
Mortality per 1000 PY
0
20
40
60
80
Age
2.0
1.5
1.0
0.5
RR
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5
10
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Duration
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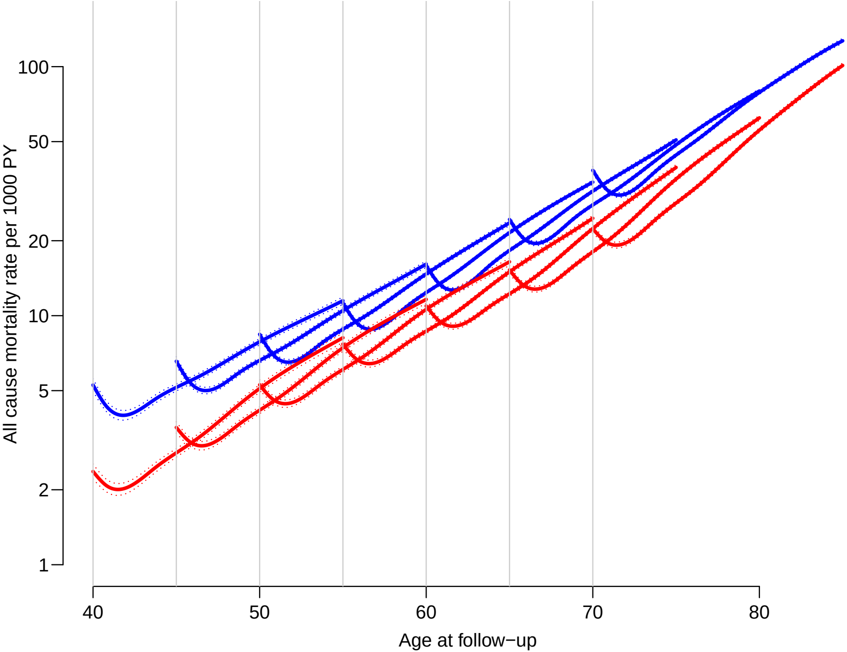
- show select fitted values to illustrate the actual effects (and their relative size)


All cause mortality rate per 1000 PY
100
50
20
10
5
2
1
40
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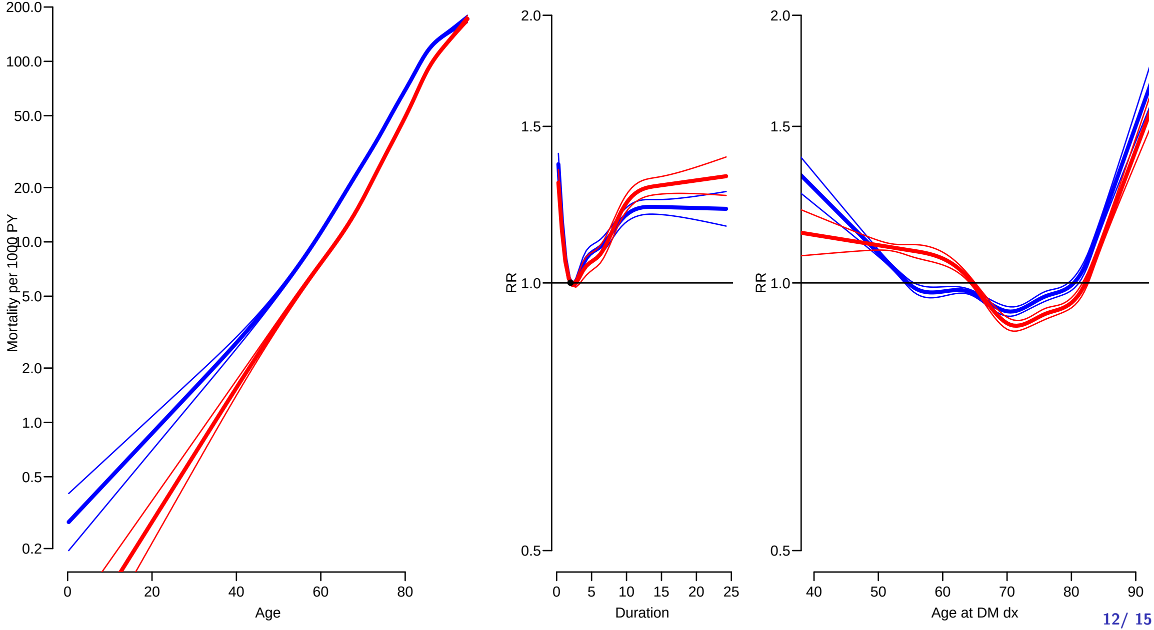
Age at follow−up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- ... which is a bit of a detour from Cox-type models.

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