Splitting the follow-up C&H 6

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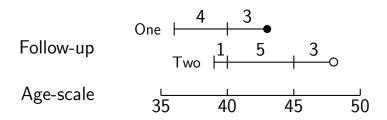
PhD-course in Epidemiology, Department of Biostatistics, Tuesday 23 March 2015

Stratification by age

If follow-up is rather short, age at entry is OK for age-stratification.

If follow-up is long, use stratification by categories of **current age**, both for:

No. of events, D, and Risk time, Y.



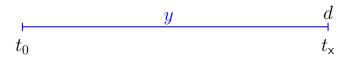
Representation of follow-up data

In a cohort study we have records of: **Events** and **Risk time**.

Follow-up data for each individual must have (at least) three variables:

- Date of entry entry date variable.
- ▶ Date of exit exit date variable
- Status at exit fail indicator-variable (0/1)

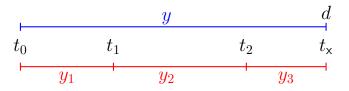
Specific for each *type* of outcome.



 $P(d \text{ at } t_{\mathsf{x}}| \mathsf{entry}\ t_0)$

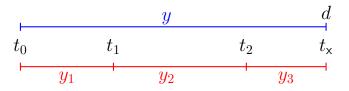
log-Likelihood

 $d\log(\lambda) - \lambda y$



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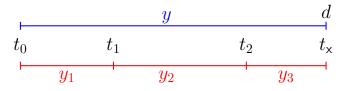
$$d\log(\lambda) - \lambda y$$



$$P(d \text{ at } t_{\mathsf{x}} | \mathsf{entry} \ t_0)$$

$$d\log(\lambda) - \lambda y$$

$$= P(\mathsf{surv}\ t_0 \to t_1 | \mathsf{entry}\ t_0)$$

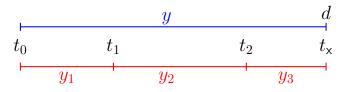


log-Likelihood

$$P(d \text{ at } t_{\mathsf{x}}|\text{entry } t_0)$$

 $d\log(\lambda) - \lambda y$

= P(surv
$$t_0 \rightarrow t_1 | \text{entry } t_0$$
)
 \times P(surv $t_1 \rightarrow t_2 | \text{entry } t_1$)



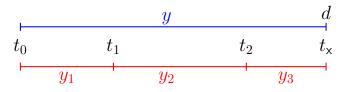
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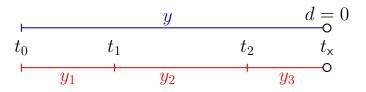
$$\times P(d \text{ at } t_{\mathsf{x}}| \mathsf{entry}\ t_2)$$

$$d\log(\lambda) - \lambda y$$

$$= 0\log(\lambda) - \lambda y_1$$

$$+0\log(\lambda) - \lambda y_2$$

$$+d\log(\lambda) - \lambda y_3$$



$$P(\mathsf{surv}\ t_0 \to t_{\mathsf{x}}|\mathsf{entry}\ t_0)$$

$$= P(\mathsf{surv}\ t_0 \to t_1 | \mathsf{entry}\ t_0)$$

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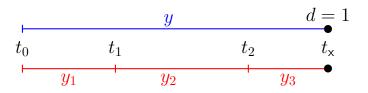
$$\times P(\mathsf{surv}\ t_2 \to t_\mathsf{x} | \mathsf{entry}\ t_2)$$

$$0\log(\lambda) - \lambda y$$

$$=0\log(\lambda)-\lambda y_1$$

$$+0\log(\lambda) - \lambda y_2$$

$$+0\log(\lambda) - \lambda y_3$$



P(event at t_x |entry t_0)

$$=\mathrm{P}(\mathsf{surv}\ t_0 o t_1 | \mathsf{entry}\ t_0)$$

$$\times P(\mathsf{surv}\ t_1 \to t_2 | \mathsf{entry}\ t_1)$$

$$\times P(\text{event at } t_{\mathsf{x}}|\text{entry } t_2)$$

$$1\log(\lambda) - \lambda y$$

$$= 0\log(\lambda) - \lambda y_1$$

$$+0\log(\lambda) - \lambda y_2$$

$$+1\log(\lambda) - \lambda y_3$$

Compute rates in different bands of:

- Compute rates in different bands of:
 - age

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 - calendar time

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 - disease duration

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- Compute rates in different bands of:
 - age
 - calendar time
 - disease duration
- Allow rates to vary along the timescale:

$$0\log(\lambda) - \lambda y_1 + 0\log(\lambda) - \lambda y_2 \rightarrow + d\log(\lambda) - \lambda y_3$$

- Compute rates in different bands of:
 - age
 - calendar time
 - disease duration
- Allow rates to vary along the timescale:

$$0\log(\lambda) - \lambda y_1 \qquad 0\log(\lambda_1) - \lambda_1 y_1 + 0\log(\lambda) - \lambda y_2 \rightarrow + 0\log(\lambda_2) - \lambda_2 y_2 + d\log(\lambda) - \lambda y_3 \qquad + d\log(\lambda_3) - \lambda_3 y_3$$

Prerequisites of splitting time

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Origin: The date where the time scale is 0:

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- ▶ Disease duration 0 at date of diagnosis
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Intervals: How should it be subdivided:

- ▶ 1-year classes? 5-year classes?
- Equal length not necessarily.

Cohort with 3 persons:

```
Id Bdate Entry Exit St
1 14/07/52 04/08/65 27/06/97 1
2 01/04/54 08/09/72 23/05/95 0
3 10/06/87 23/12/91 24/07/98 1
```

- ▶ Define strata: 10-years intervals of current age.
- Split Y for every subject accordingly
- Treat each segment as a separate unit of observation.
- Keep track of exit status in each interval.

Splitting the follow up

	subj. 1	subj. 2	subj. 3
Age at E ntry:	13.06	18.44	4.54
Age at e X it:	44.95	41.14	11.12
S tatus at exit:	Dead	Alive	Dead
\overline{Y}	31.89	22.70	6.58
D	1	0	1

Where did the pieces go?

	subj. 1		subj.	subj. 2		subj. 3		\sum	
Age	Y	D	\overline{Y}	D	\overline{Y}	D	\overline{Y}	D	
0-	0.00	0	0.00	0	5.46	0	5.46	0	
10-	6.94	0	1.56	0	1.12	1	8.62	1	
20-	10.00	0	10.00	0	0.00	0	20.00	0	
30-	10.00	0	10.00	0	0.00	0	20.00	0	
40-	4.95	1	1.14	0	0.00	0	6.09	1	
\sum	31.89	1	22.70	0	6.58	1	60.17	2	

Time-splitting with SAS: %Lexis

```
%Lexis( data=a, entry=Entry, exit=Exit, fail=St, origin=bdate, scale=365.25, breaks=0 to 80 by 10 ) ;
```

id	Bdate	Entry	Exit	St	risk	left
1 1 1 1 2 2 2 2	14/07/1952 14/07/1952 14/07/1952 14/07/1952 01/04/1954 01/04/1954 01/04/1954 01/04/1954	03/08/1965 14/07/1972 14/07/1982 14/07/1992 08/09/1972 01/04/1974 31/03/1984 01/04/1994	14/07/1972 14/07/1982 14/07/1992 27/06/1997 01/04/1974 31/03/1984 01/04/1994 23/05/1995	0 0 0 1 0 0	6.9432 10.0000 10.0000 4.9528 1.5606 10.0000 10.0000 1.1417	10 20 30 40 10 20 30 40
3 3	10/06/1987 10/06/1987	23/12/1991 09/06/1997	09/06/1997 24/07/1998	0 1	5.4634 1.1211	0 10

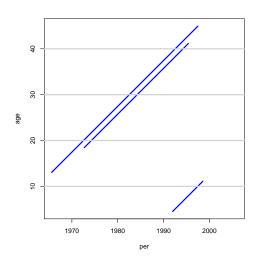
Time-splitting with Stata stset, stsplit

Time-splitting with R Lexis, splitLexis

```
library( Epi )
Lx <- Lexis( entry = list( per = Entry,</pre>
                         age = Entry-Bdate ),
             exit = list( per = Exit ),
      exit.status = factor(St, labels=c("Alive", "Dead")),
             data = coh )
Ls <- splitLexis( Lx, breaks=seq(0,100,10), time.scale="age" )
               age lex.dur lex.Cst lex.Xst Id
lex.id
           per
                                                  Bdate
                                                           F.n
     1 1965.589 13.056
                       6.943
                               Alive
                                       Alive
                                             1 1952.533 1965.
     1 1972.533 20.000
                      10.000 Alive
                                      Alive
                                             1 1952.533 1965.
     1 1982.533 30.000 10.000 Alive Alive
                                             1 1952.533 1965.
     1 1992.533 40.000 4.952 Alive Dead
                                             1 1952,533 1965.
    2 1972.686 18.439 1.560 Alive
                                             2 1954.246 1972.
                                      Alive
    2 1974.246 20.000
                       10.000 Alive
                                       Alive
                                             2 1954.246 1972.
    2 1984,246 30,000
                       10.000 Alive
                                       Alive
                                             2 1954, 246 1972.
    2 1994.246 40.000 1.141 Alive
                                      Alive
                                             2 1954, 246 1972.
    3 1991.974 4.536 5.463 Alive
                                      Alive
                                             3 1987.437 1991.
    3 1997.437 10.000 1.121 Alive
                                      Dead
                                             3 1987.437 1991.
```

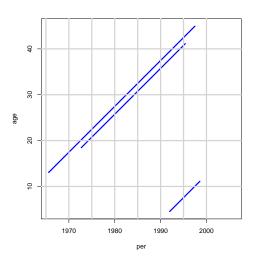
Time-splitting with R Lexis, splitLexis

plot(Ls, col="blue", lwd=3)



Time-splitting with R Lexis, splitLexis

Ls <- splitLexis(Ls, breaks=seq(1900,2000,5), time.scale="per" plot(Ls, col="blue", lwd=3)



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- Possibility of different rates in different intervals.

Data for Cox-regression has only one record per person.

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- Additional time-scales require multiple records per person

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For each little piece of follow up we attach the relevant covariates:

- Fixed covariates. (sex, genotype, . . .)
- Deterministically time-varying covariates:
 age, time since entry, calendar time all derived from the current date.
- Non-deterministically varying covariates. (current smoking habits, occupational exposure, . . .)

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Among the covariates are some that model the time-effect (in the IHD-example, age).

Relation to the Cox-model:

$$\lambda(t) = \lambda_0(t) \exp(x_1 \beta_1 + x_2 \beta_2 + \cdots)$$

$$= \exp(\log(\lambda_0(t)) + x_1 \beta_1 + x_2 \beta_2 + \cdots)$$

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The baseline hazard — unspecified in the Cox-model — is replaced by a parmetric function, $\exp(z_1\alpha_1 + z_2\alpha_2 + \cdots)$

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The likelihood contribution from one person is a **product** of **conditional** probabilities.

Because the likelihood is a **product**, we can use the program (proc genmod, glm, ...) as if they were independent; we are only interested in getting the maximum likelihood estimates.

The offset

Need to take account of the "covariate" log(Y), which has a regression coefficient fixed to be one:

$$\log(\lambda Y) = x_1 \beta_1 + x_2 \beta_2 + \dots + \log(Y)$$

log(Y) is called an **offset**-variable.

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 - timescales (age, calendar time, time since entry)
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- Model rates using the covariates in proc genmod
- ▶ Note: there is no difference in how time-scales and other covariates are treated in the model.

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- ▶ The log-likelihood contribution from those with the same lambda is $\sum D\log\lambda$ the same as from aggregated data.
- ► The log-likelihood is the same for split data and aggregated data no need to tabulate first.

Your turn now: IHD data

The following exercise is designed to illustrate how follow-up time is subdivided in order to produce the table of events and person-years. Furthermore the aim is to show you that tabulated data and time-split data gives the same results if only age and exposure are used as variables.

We will first analyze frequency records as above (these are almost identical to Table 22.6 in C & H). Next, we shall read the individual records and construct the corresponding table of cases and person-years.

- Import the program ihd-lexis.sas to the program editor. Run the first part of the program — the part reading the tabulated data and proc genmod. Compare with the results from table xx in Clayton & Hills.
- 2. Next, use the second part of the program to read the individual records from the file diet.txt, including the proc print and check on the output that it looks reasonable and that you understand what the data represents.

Time-splitting with SAS I

3. Now you should import the macro %Lexis and use it to split into the age intervals 40–50, 50–60 and 60–70 years:

In order to use this you must first load it from the appropriate folder folder on the net:

```
* This will list the included code in your log-window; options source2; filename lexispr url "http://www.biostat.ku.dk/~bxc/Lexis/Lexis.sas"; %inc lexispr;
```

Once you have specified %inc lexispr; and run that line in SAS, SAS will know the macro %lexis and you can use it in the rest of the session.

Time-splitting with SAS II

4. The time-splitting is now done by running the SAS-macro "Lexis A SAS-macro is a piece of SAS-program (normally quite long) where certain small parts of the program can be changed when the program is run. The SAS-convention is that names of such programs start with a "%".

To use the macro we must specify the follow-up information from the input file:

- Date of entry into the study doe
- ▶ Date of exit from the study dox
- Status at exit from the study chd (1 if CHD occurred at dox, 0 otherwise).

Moreover, we must decide which timescale to split the data on. In this case we want to split along the scale "current age", i.e. time since date of birth.

Time-splitting with SAS III

5. To this end we must specify:

- The origin of the time-scale, i.e. where the time-scale is 0, in this case date of birth — dob.
- ► The intervals where we want the follow-up grouped, here ages 40–50, 50–60 and 60–70.
- As a purely technical thing we need to specify the conversion between the scale in which time is measured in the input dataset (in this case days) and in the specification of the grouping (in this case years) — 365.25.

In the case of %Lexis we must supply these 6 parameters in order to specify how to split time.

Finally we must tell the program where the original data is, where the time-split data has to go, and what the name of the age-variable should be.

Time-splitting with SAS IV

This looks like this (you do not have to write the stuff between the /*...*/):

```
%Lexis( data = ihdindiv, /* Dataset with original data
       out = ihdsplit,
                           /* Dataset with time-split data
                                                              */
       entry = doe,
                            /* Date of entry
       exit = dox,
                           /* Date of exit
                                                              */
                      /* Event (failure) indicator
       fail = chd,
       breaks = 40 to 70 by 10, /* Where to split the time scale
                                                              */
       origin = dob,
                            /* Origin of the time-scale
                                                              */
       scale = 365.25,
                             /* Conversion from days to years
                                                              */
       left = agr );
                             /* The name of the new age-variable */
```

Run this piece of SAS code.

```
(In the top of the file
```

http://www.biostat.ku.dk/~bxc/Lexis/Lexis.sas are some more detailed explanations of how to use %Lexis).

Tabulation of time-split data with SAS I

- 6. How many records are in the resulting dataset (ihdsplit)
- 7. Take at look at the resulting data file, for example the first 20 records:

```
proc print data = ihdsplit (obs=20) ;
run ;
```

How does this compare with the the original dataset?

8. Use %PYtab to tabulate IHD-cases and person-years by exposure and age-group. You must first get this from the net as you did with the %Lexis macro:

```
filename pytabpr url
  "http://www.biostat.ku.dk/~bxc/Lexis/PYtab.sas";
%inc pytabpr ;
```

Tabulation of time-split data with SAS II

Once you have imported the macro you can use it:

Compare with the sums from the table given in the first data step in ihd-lexis.sas