

Representation and prediction in multistate models

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Representation of FU

- ▶ Many different ways of representing follow-up:
 - ▶ msm
 - ▶ mstate
 - ▶ etm
- ▶ some dedicated to a special model
- ▶ mostly on one timescale (the `st` machinery in Stata)
- ▶ generally derived from follow-up in a clinical trial with
 - ▶ entry at 0
 - ▶ exit at t
 - ▶ risk time t

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MSM representation

Epochs (time-intervals)

- ▶ **Time** at start: The value of the timescales of interest.
- ▶ **Length** of the epoch — risk time ("exposure")
- ▶ **State** in which the the time is spent
- ▶ **next state**: the state the person moves on to

This is a representation of the **risk time**

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Representation of likelihood

- ▶ The likelihood terms in a multistate model comes naturally as one per **transition**:
- ▶
$$\exp \left(\int_{\text{in}}^{\text{out}} \lambda_j(u) du \right)$$
- ▶ ... and for the events, additionally $\lambda_j(\text{out})$
- ▶ This is the **stacked** representation, allowing joint modeling of all transition rates in one go:
- ▶ Separate or joint parameters for each transition.

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Lexis objects

`data.frame` with a few extra features:
`timescales` — chose any number, name as you please
`lex.id` — person identification
`lex.dur` — length of the epoch represented
`lex.Cst` — state of risk time (`Current state`)
`lex.Xst` — neXt state

Lexis objects - split objects

Splitting time with `splitLexis`:

- ▶ risk time in many small intervals
- ▶ assuming constant rates in each
- ▶ allows simple parametric modeling with `glm`
- ▶ using any number of time scales
- ▶ prediction of rates
- ▶ calculation of any derived measure

glm modeling form object Lx

```
> w2i <- glm( lex.Xst=="Ill" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- glm( lex.Xst=="Dead" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> i2d <- glm( lex.Xst=="Dead" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Ill" ) )
```

coxph modeling form object Lx

```
> w2i <- coxph( Surv( time, time+lex.dur, lex.Xst=="Ill" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- coxph( Surv( time, time+lex.dur, lex.Xst=="Dead" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Well" ) )
> #
> i2d <- coxph( Surv( time, time+lex.dur, lex.Xst=="Dead" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Ill" ) )
```

glm modeling form object Lx with update

```
> w2i <- glm( lex.Xst=="Ill" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- update( w2i, lex.Xst=="Dead" ~ . )
> #
> i2d <- update( w2d, data = subset( Lx, lex.Cst=="Ill" ) )
```

— but if you really are using the same covariates:

```
> mA <- glm( lex.Fail ~ lex.Tr + (v1 + v2 + v3):lex.Tr,
+               family=poisson, offset=log(lex.dur),
+               data = stack.Lexis( Lx ) )
```

lex.Tr has levels "Well->Ill", "Well->Dead" and "Ill->Dead"

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Stacking – interface to mstate

- ▶ Stacking of the follow-up gives the terms that make up the likelihood
- ▶ lex.Tr has levels "Well->Ill", "Well->Dead" and "Ill->Dead"
- ▶ lex.Fail is logical.
- ▶ Used for the interface function msdata.Lexis
 - stacks the data and shaves away features.

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Utilities — summaries

- ▶ summary — gives table of transitions and risk time
- ▶ boxes — shows it in a nice transition diagram (boxes)
- ▶ plot — plots follow-up in a Lexis diagram

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Utilities — manipulation & restructuring

- ▶ cutLexis — cuts follow-up at a time point; transition to a new state.
- ▶ splitLexis — splits time in small intervals along a time scale at designated breaks.
- ▶ Both features need stsplit in Stata (+plus a bit of fidgeting)

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Predicting using a single time scale

- ▶ With a parametric model for rates:
- ▶ closed form expressions for state probabilities etc.
- ▶ wrapping in a function with parameters as arguments
- ▶ parametric bootstrap does all for you
- ▶ multistate machinery from Michael C

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Prediction using realistic models

- ▶ Simulation of an entire dataset through MS model
 - ▶ allowing multiple time scale
 - ▶ ... also "time since state S"
- ▶ Enumeration of persons in states gives
 - state occupancy probabilities
 - expected sojourn times
 - expected life time probabilities

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Diabetologia
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ARTICLE

Years of life gained by multifactorial intervention in patients with type 2 diabetes mellitus and microalbuminuria: 21 years follow-up on the Steno-2 randomised trial

Peter Gade^{1,2} · Jens Oelgaard^{1,2,3} · Bendix Carstensen³ · Peter Rossing^{3,4,5} · Henrik Lund-Andersen^{3,5,6} · Hans-Henrik Parving^{5,7} · Oluf Pedersen⁸

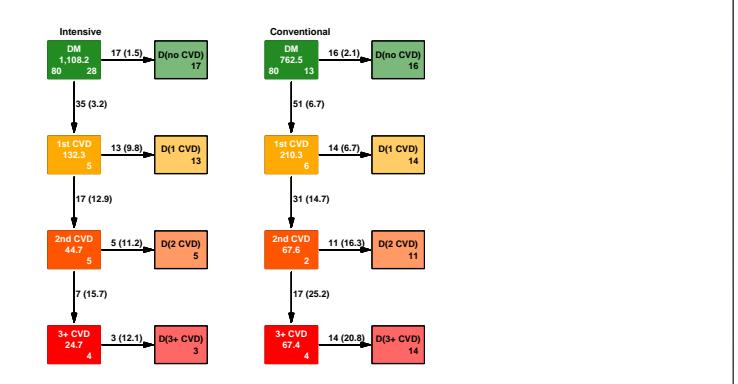
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Abstract

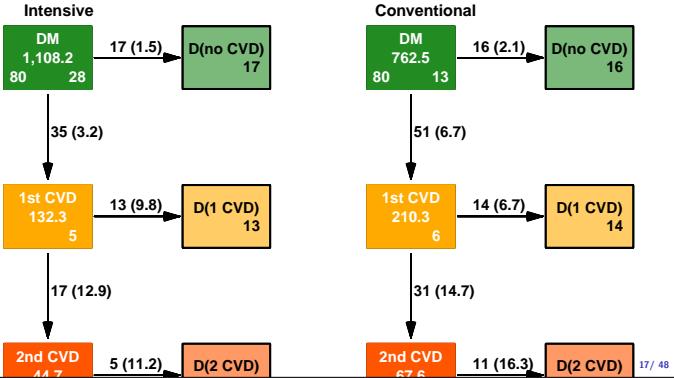
Aims/hypothesis The aim of this work was to study the potential long-term impact of a 7.8 years intensified multifactorial

pharmacological approaches. After 7.8 years the study continued as an observational follow-up with all patients receiving treatment as for the original intensive therapy group. The re-

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Expected lifetime and YLL (well, gained)

Expected lifetime (years) in the Steno 2 cohort during the first 20 years after baseline by treatment group and CVD status.

State	Intensive	Conventional	Int.–Conv.
Alive	15.6	14.1	1.5
No CVD	12.7	10.0	2.6
Any CVD	3.0	4.1	-1.1

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Hazard ratios

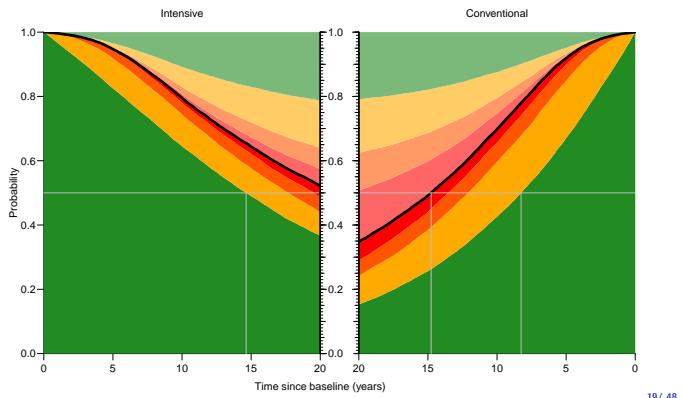
HR, Int. vs. Conv.	CVD event		Mortality
	0.55 (0.39;0.77)	0.83 (0.54; 1.30)	
	p=0.438	p=0.261	
H_0 : HR = 1	p=0.425	p=0.001	
HR vs. 0 CVD events:			
0 (ref.)	1.00	1.00	
1	2.43 (1.67;3.52)	3.08 (1.82; 5.19)	
2	3.48 (2.15;5.64)	4.42 (2.36; 8.29)	
3+		7.76 (4.11;14.65)	

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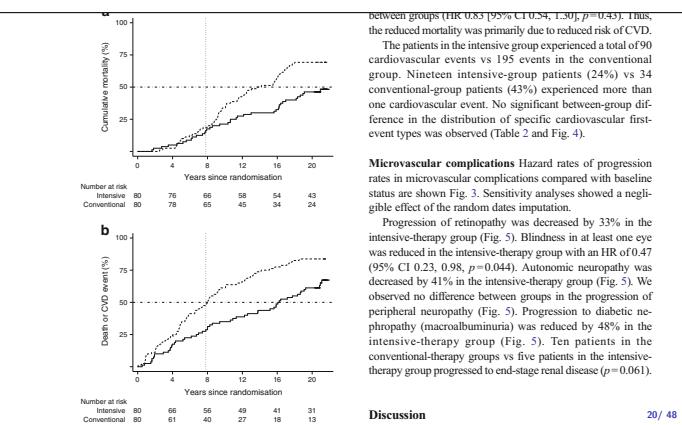
Expected lifetime (years) during the first 20 years after baseline by sex, age, treatment group and CVD status.

sex	Men			Women			
	state	age	Int.	Conv.	Int.–Conv.	Int.	Conv.
Alive	45	18.5	17.5	1.0	19.1	18.4	0.7
	50	17.2	16.1	1.1	18.0	17.2	0.8
	55	15.6	13.8	1.8	17.4	15.9	1.6
	60	13.9	11.6	2.2	15.5	13.7	1.8
	65	11.2	9.5	1.8	13.3	11.4	2.0
No CVD	45	14.9	12.5	2.4	15.8	14.3	1.5
	50	14.0	11.1	2.9	15.1	12.9	2.2
	55	12.2	9.7	2.5	14.3	11.6	2.7
	60	10.9	8.2	2.7	12.4	9.9	2.6
	65	9.0	6.7	2.2	10.7	8.3	2.4

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Multistate models in practice:

- ▶ Representation:
 - ▶ States
 - ▶ Transitions
 - ▶ Sojourn times
 - ▶ Rates
- ▶ Analysis of rates:
 - ▶ Cox-model
 - ▶ Poisson model
- ▶ Reporting
 - ▶ Rates
 - ▶ HRs
 - ▶ Probabilities
 - ▶ Expected lifetime

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Representation of multistate FU: Lexis

- ▶ Allowing multiple time scales
 - ▶ time-scale variables — the starting point on each time scale
 - ▶ sojourn time variable `lex.dur` — risetime, exposure
 - ▶ state variables:
- ▶ Allowing multiple states
 - ▶ `lex.Cst` — the state in which follow-up (`lex.dur`) occurs
 - ▶ `lex.Xst` — the state in which

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```
> head( st2[,-c(1,4:5,12,13) ] )
  allocation sex doBth doDM doBase doCVD1 doCVD2 doCVD3 doDth
1  Intensive M 1932.300 1991.093 1993.357 2014.457 NA NA NA
2  Intensive M 1946.813 1982.130 1993.357 2001.231 2009.570 2010.162 NA
3 Conventional M 1943.377 1982.560 1993.367 2001.639 NA NA 2001.639
4 Conventional M 1944.777 1976.791 1993.367 1995.415 1997.461 2003.443 2003.443
5 Conventional M 1935.993 1986.346 1993.162 1993.959 1994.657 1998.047 1998.047
6 Conventional M 1946.942 1986.404 1993.201 1998.769 NA NA NA
```

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```
> L1 <- cutLexis( L0, cut = L0$doCVD1,
+                     new.state = "1st CVD",
+                     precursor.states = c("DM"),
+                     split.states = TRUE,
+                     new.scale = "tfCVD")
> #
> L1 <- cutLexis( L1, cut = L1$doCVD2,
+                     new.state = "2nd CVD",
+                     precursor.states = c("DM", "1st CVD"),
+                     split.states = TRUE)
> #
> L1 <- cutLexis( L1, cut = L1$doCVD3,
+                     new.state = "3rd CVD",
+                     precursor.states = c("DM", "1st CVD", "2nd CVD"),
+                     split.states = TRUE)
> levels( L1 )
[1] "DM"                               "1st CVD"
[3] "2nd CVD"                          "3rd CVD"
[5] "Dead"                             "Dead(1st CVD)"
[7] "Dead(1st CVD)(2nd CVD)"          "Dead(1st CVD)(2nd CVD)(3rd CVD)"

> levels( L1$lex.Cst )[5:8] <-
+ levels( L1$lex.Xst )[5:8] <- paste("Dth(",0:3,")",sep="")
```

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```
> L2 <- Lexis( entry = list( per = doBase,
+                           age = doBase-doBth,
+                           dur = doBase-doDM,
+                           tsb = 0 ),
+                           exit = list( per = doEnd ),
+                           exit.status = factor( ( !is.na(doDth) ) + deathCVD,
+                           labels=c("DM","D-oth","CV-D") ),
+                           data = st2 )

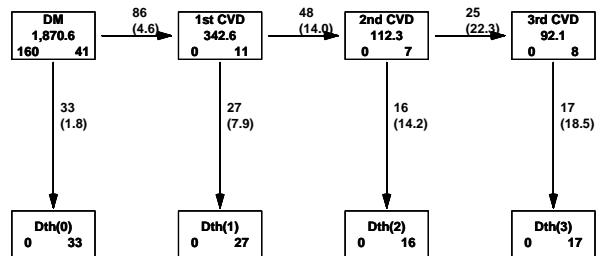
NOTE: entry.status has been set to "DM" for all.
```

> summary(L2)

Transitions:
To
From DM D-oth CV-D Records: Events: Risk time: Persons:
DM 67 55 38 160 93 2417.6 160

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```
> boxes( L1, boxpos=list( x=c(rep(seq(15,85,,4),2) ),
+                         y=rep(c(85,20),each=4) ), show.BE=TRUE, scale.R=100 )
```



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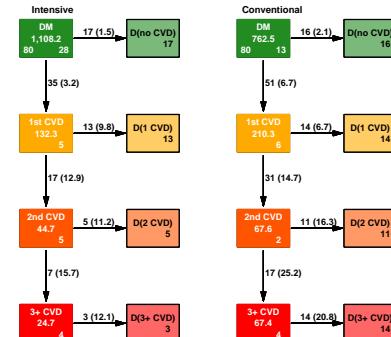
```
> L0 <- Lexis( entry = list( per = doBase,
+                           age = doBase-doBth,
+                           dur = doBase-doDM,
+                           tsb = 0 ),
+                           exit = list( per = doEnd ),
+                           exit.status = factor( ( !is.na(doDth) ) ,
+                           labels=c("DM","Dead") ),
+                           data = st2 )

NOTE: entry.status has been set to "DM" for all.
```

> summary(L0)

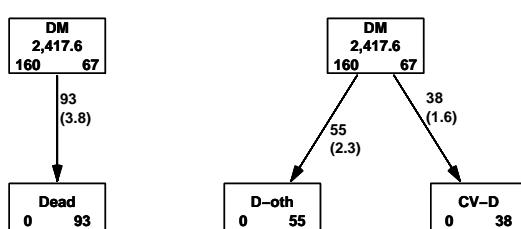
Transitions:
To
From DM Dead Records: Events: Risk time: Persons:
DM 67 93 160 93 2417.6 160

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```
> par( mflow=c(1,2) )
> boxes( L0, boxpos=list(x=c(50,50),y=c(85,15)),show.BE=T,scale.R=100 )
> boxes( L2, boxpos=list(x=c(50,15,85),y=c(85,15,15)),show.BE=T,scale.R=100)
```



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An added time scale

We added the timescale tfCVD using new.scale="tfCVD"

```
> subset( L1, lex.id==5 )[,1:9]
  per    age    dur    tsb    tfCVD   lex.dur lex.Cst lex.Xst lex.id
10 1993.162 57.16906 6.815916 0.0000000      NA 0.7967146  DM 1st CVD 5
11 1993.959 57.96578 7.612631 0.7967146 0.0000000 0.6981520 1st CVD 2nd CVD 5
12 1994.657 58.66393 8.310783 1.4948665 0.6981520 3.3894593 2nd CVD Dth(2) 5
```

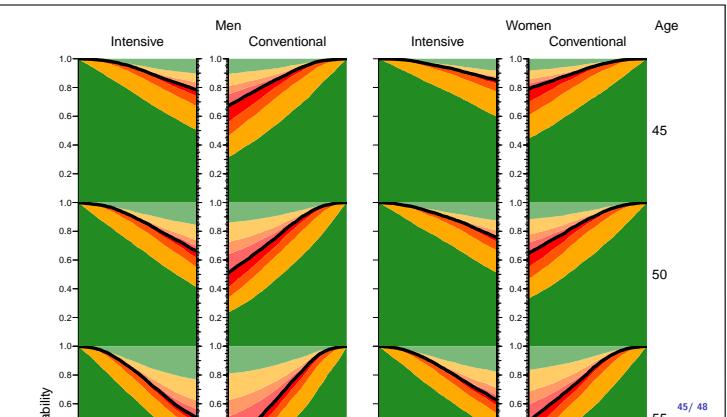
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Simulation: baseline (init) object

```
> St2 <- subset( L1, select = c(timeScales(L1),"lex.Cst","lex.dur",
+ "allocation","sex" ) )
> levels(St2$lex.Cst) <- levels(S1$lex.Cst)
> nr <- nrow(St2)
> St2[1:nr,"tsb"] <- 0
> St2[1:nr,"lex.Cst"] <- levels(St2$lex.Cst)[1]
> St2[1:nr,"allocation"] <- levels(S1$allocation)[1]
> St2 <- rbind( St2, St2 )
> St2[1:nr,"allocation"] <- levels(S1$allocation)[2]
```

St2 is a copy of the original date with allocation="Intensive" plus a copy of the original date with allocation="Conventional"

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Age

50

45

Simulating a follow up dataset

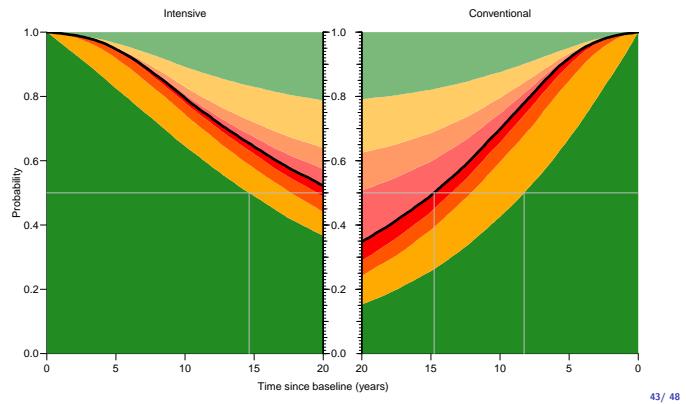
```
> sim2 <- simLexis( Tx = TM1,
+                     init = St2,
+                     N = 100,
+                     time pts = seq(0,21,0.1) )
> StC <- nState( subset(sim2,allocation=="Conventional"),
+                  at=seq(0,20,0.1), from=0, time.scale="tsb" )
> StI <- nState( subset(sim2,allocation=="Intensive"),
+                  at=seq(0,20,0.1), from=0, time.scale="tsb" )
> prm <- c(1:4,8:5)
> PC <- pState( StC, perm=prm )
> PI <- pState( StI, perm=prm )
```

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History

- ▶ Epi package grew out of "Statistical Practice in Epidemiology with R", annually since 2002 in Tartu Estonia
- ▶ Lexis machinery conceived by Martyn Plummer, IARC
- ▶ Naming originally by David Clayton & Michael Hills, stlexis in Stata, later renamed stssplit
- ▶ David Clayton wrote a lexis function for the Epi package. Obsolete now.

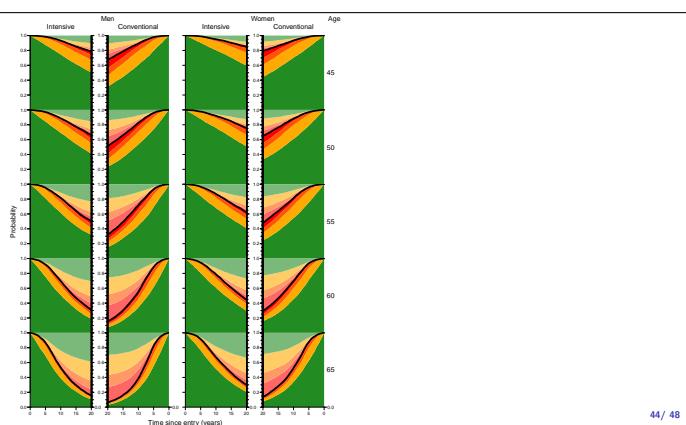
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Summary

- ▶ Proper representation of multistate data essential: States, transitions, risk time
- ▶ Readable modeling code
- ▶ Calculation of state probabilities requires a simulation in any realistic situation
- ▶ Examples of use in:
<http://bendixcarstensen.com/AdvCoh/Lexis-ex/>

Thanks for your attention

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