

Representation and prediction in multistate models

Bendix Carstensen Steno Diabetes Center,
Gentofte, Denmark
& Department of Biostatistics, University of Copenhagen
b@bxc.dk
<http://BendixCarstensen.com>

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1/ 48

6/ 48

Representation of FU

- ▶ Many different ways of representing follow-up:
 - ▶ msm
 - ▶ mstate
 - ▶ etm
- ▶ some dedicated to a special model
- ▶ mostly on one timescale (the `st` machinery in Stata)
- ▶ generally derived from follow-up in a clinical trial with
 - ▶ entry at 0
 - ▶ exit at t
 - ▶ risk time t

2/ 48

7/ 48

MSM representation

Epochs (time-intervals)

- ▶ **Time** at start: The value of the timescales of interest.
- ▶ **Length** of the epoch — risk time ("exposure")
- ▶ **State** in which the the time is spent
- ▶ **next state**: the state the person moves on to

This is a representation of the **risk time**

3/ 48

8/ 48

Representation of likelihood

- ▶ The likelihood terms in a multistate model comes naturally as one per **transition**:
- ▶
$$\exp \left(\int_{\text{in}}^{\text{out}} \lambda_j(u) du \right)$$
- ▶ ... and for the events, additionally $\lambda_j(\text{out})$
- ▶ This is the **stacked** representation, allowing joint modeling of all transition rates in one go:
- ▶ Separate or joint parameters for each transition.

4/ 48

9/ 48

Lexis objects

`data.frame` with a few extra features:

`timescales` — chose any number, name as you please
`lex.id` — person identification
`lex.dur` — length of the epoch represented
`lex.Cst` — state of risk time (**Current state**)
`lex.Xst` — **neXt state**

Lexis objects - split objects

Splitting time with `splitLexis`:

- ▶ risk time in many small intervals
- ▶ assuming constant rates in each
- ▶ allows simple parametric modeling with `glm`
- ▶ using any number of time scales
- ▶ prediction of rates
- ▶ calculation of any derived measure

glm modeling form object Lx

```
> w2i <- glm( lex.Xst=="Ill" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- glm( lex.Xst=="Dead" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> i2d <- glm( lex.Xst=="Dead" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Ill" ) )
```

coxph modeling form object Lx

```
> w2i <- coxph( Surv( time, time+lex.dur, lex.Xst=="Ill" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- coxph( Surv( time, time+lex.dur, lex.Xst=="Dead" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Well" ) )
> #
> i2d <- coxph( Surv( time, time+lex.dur, lex.Xst=="Dead" ) ~ v1 + v2 + v3,
+                 data = subset( Lx, lex.Cst=="Ill" ) )
```

glm modeling form object Lx with update

```
> w2i <- glm( lex.Xst=="Ill" ~ v1 + v2 + v3,
+               family=poisson, offset=log(lex.dur),
+               data = subset( Lx, lex.Cst=="Well" ) )
> #
> w2d <- update( w2i, lex.Xst=="Dead" ~ . )
> #
> i2d <- update( w2d, data = subset( Lx, lex.Cst=="Ill" ) )
```

— but if you really **are** using the same covariates:

```
> mA <- glm( lex.Fail ~ lex.Tr + (v1 + v2 + v3):lex.Tr,
+               family=poisson, offset=log(lex.dur),
+               data = stack.Lexis( Lx ) )
```

`lex.Tr` has levels "Well->Ill", "Well->Dead" and "Ill->Dead"

Stacking – interface to mstate

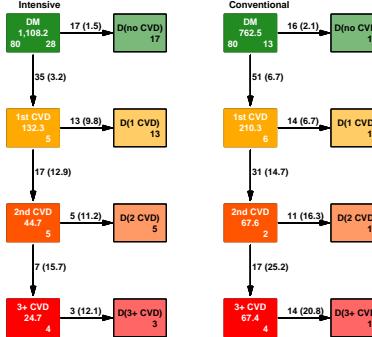
- ▶ Stacking of the follow-up gives the terms that make up the likelihood
- ▶ `lex.Tr` has levels "Well->Ill", "Well->Dead" and "Ill->Dead"
- ▶ `lex.Fail` is logical.
- ▶ Used for the interface function `msdata.Lexis`
 - stacks the data and shaves away features.

5/ 48

10/ 48

Utilities — summaries

- ▶ **summary** — gives table of transitions and risk time
- ▶ **boxes** — shows it in a nice transition diagram (boxes)
- ▶ **plot** — plots follow-up in a Lexis diagram

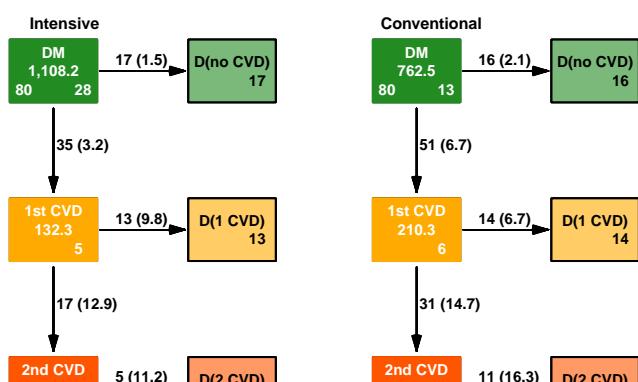


11 / 48

16 / 48

Utilities — manipulation & restructuring

- ▶ **cutLexis** — cuts follow-up at a time point; transition to a new state.
- ▶ **splitLexis** — splits time in small intervals along a time scale at designated breaks.
- ▶ Both features need **stssplit** in Stata (+plus a bit of fidgeting)



12 / 48

17 / 48

Predicting using a single time scale

- ▶ With a parametric model for rates:
- ▶ closed form expressions for state probabilities etc.
- ▶ wrapping in a function with parameters as arguments
- ▶ parametric bootstrap does all for you
- ▶ multistate machinery from Michael C

Hazard ratios

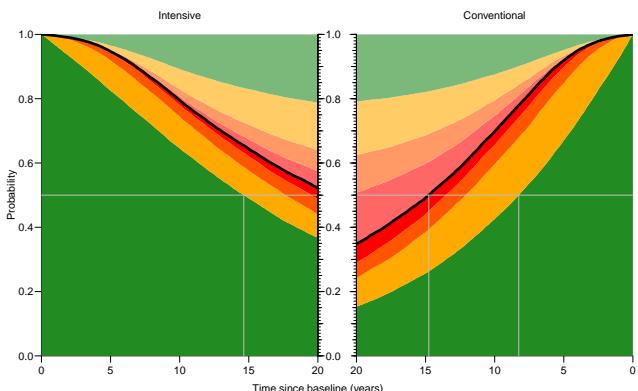
	CVD event	Mortality
HR, Int. vs. Conv.	0.55 (0.39;0.77)	0.83 (0.54; 1.30)
H ₀ : PH btw. CVD groups	p=0.438	p=0.261
H ₀ : HR = 1	p=0.425	p=0.001
HR vs. 0 CVD events:		
0 (ref.)	1.00	1.00
1	2.43 (1.67;3.52)	3.08 (1.82; 5.19)
2	3.48 (2.15;5.64)	4.42 (2.36; 8.29)
3+	7.76 (4.11;14.65)	

13 / 48

18 / 48

Prediction using realistic models

- ▶ Simulation of an entire dataset through MS model
 - ▶ allowing multiple time scale
 - ▶ ... also "time since state S"
- ▶ Enumeration of persons in states gives
 - state occupancy probabilities
 - expected sojourn times
 - expected life time probabilities



14 / 48

19 / 48

Years of life gained by multifactorial intervention in patients with type 2 diabetes mellitus and microalbuminuria: 21 years follow-up on the Steno-2 randomised trial

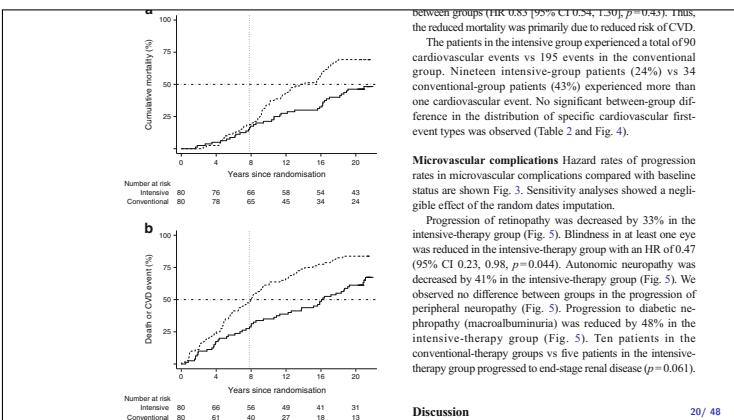
Peter Gøde^{1,2} · Jens Oelgaard^{1,2,3} · Bendix Carstensen³ · Peter Rossing^{3,4,5} ·
Henrik Lund-Andersen^{3,5,6} · Hans-Henrik Parving^{5,7} · Oluf Pedersen⁸

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Abstract

Aims/hypothesis The aim of this work was to study the potential long-term impact of a 7.8 years intensified multifactorial

pharmacological approaches. After 7.8 years the study continued as an observational follow-up with all patients receiving treatment as for the original intensive therapy group. The pri-



Discussion

20 / 48

Expected lifetime and YLL (well, gained)

Expected lifetime (years) in the Steno 2 cohort during the first 20 years after baseline by treatment group and CVD status.

State	Intensive	Conventional	Int.-Conv.
Alive	15.6	14.1	1.5
No CVD	12.7	10.0	2.6
Any CVD	3.0	4.1	-1.1

21 / 48

```
> L2 <- Lexis( entry = list( per = doBase,
+                               age = doBase-doBth,
+                               dur = doBase-doDM,
+                               tsb = 0 ),
+                               exit = list( per = doEnd ),
+                               exit.status = factor( ( !is.na(doDth) ) + deathCVD,
+                               labels=c("DM","D-oth","CV-D") ),
+                               data = st2 )

NOTE: entry.status has been set to "DM" for all.
```

Transitions:

```
To
From DM D-oth CV-D Records: Events: Risk time: Persons:
DM 67   55   38    160     93   2417.6    160
```

26 / 48

Expected lifetime (years) during the first 20 years after baseline by sex, age, treatment group and CVD status.

sex	Men			Women				
	state	age	Int.	Conv.	Int.-Conv.	Int.	Conv.	Int.-Conv.
Alive	45	18.5	17.5	1.0	19.1	18.4	0.7	
	50	17.2	16.1	1.1	18.0	17.2	0.8	
	55	15.6	13.8	1.8	17.4	15.9	1.6	
	60	13.9	11.6	2.2	15.5	13.7	1.8	
	65	11.2	9.5	1.8	13.3	11.4	2.0	
No CVD	45	14.9	12.5	2.4	15.8	14.3	1.5	
	50	14.0	11.1	2.9	15.1	12.9	2.2	
	55	12.2	9.7	2.5	14.3	11.6	2.7	
	60	10.9	8.2	2.7	12.4	9.9	2.6	
	65	9.0	6.7	2.2	10.7	8.3	2.4	

22 / 48

Multistate models in practice:

- ▶ Representation:
 - ▶ States
 - ▶ Transitions
 - ▶ Sojourn times
 - ▶ Rates
- ▶ Analysis of rates:
 - ▶ Cox-model
 - ▶ Poisson model
- ▶ Reporting
 - ▶ Rates
 - ▶ HRs
 - ▶ Probabilities
 - ▶ Expected lifetime

23 / 48

```
> LO <- Lexis( entry = list( per = doBase,
+                               age = doBase-doBth,
+                               dur = doBase-doDM,
+                               tsb = 0 ),
+                               exit = list( per = doEnd ),
+                               exit.status = factor( ( !is.na(doDth) ),
+                               labels=c("DM","Dead") ),
+                               data = st2 )

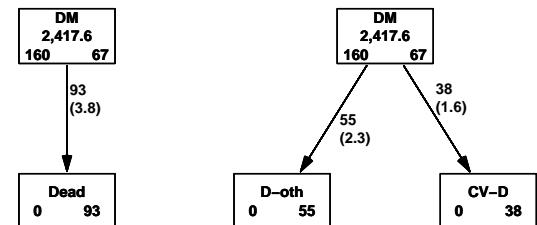
NOTE: entry.status has been set to "DM" for all.
```

Transitions:

```
To
From DM Dead Records: Events: Risk time: Persons:
DM 67   93    160     93   2417.6    160
```

27 / 48

```
> par( mflow=c(1,2) )
> boxes( LO, boxpos=list(x=c(50,50),y=c(85,15)),show.BE=T,scale.R=100)
> boxes( L2, boxpos=list(x=c(50,15,85),y=c(85,15,15)),show.BE=T,scale.R=100)
```



28 / 48

Representation of multistate FU: Lexis

- ▶ Allowing multiple time scales
 - ▶ time-scale variables — the starting point on each time scale
 - ▶ sojourn time variable `lex.dur` — riskttime, exposure
 - ▶ state variables:
- ▶ Allowing multiple states
 - ▶ `lex.Cst` — the state in which follow-up (`lex.dur`) occurs
 - ▶ `lex.Xst` — the state in which

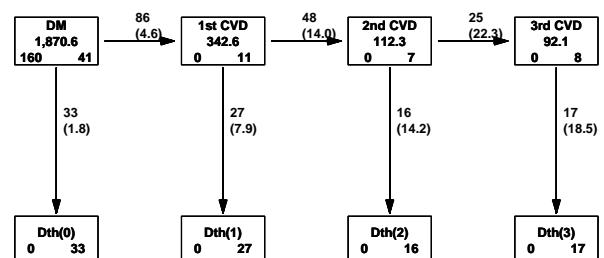
24 / 48

```
> Li <- cutLexis( LO, cut = LO$doCVD1,
+                               new.state = "1st CVD",
+                               precursor.states = c("DM"),
+                               split.states = TRUE,
+                               new.scale = "tfCVD")
> #
> Li <- cutLexis( L1, cut = L1$doCVD2,
+                               new.state = "2nd CVD",
+                               precursor.states = c("DM","1st CVD"),
+                               split.states = TRUE )
> #
> Li <- cutLexis( L1, cut = L1$doCVD3,
+                               new.state = "3rd CVD",
+                               precursor.states = c("DM","1st CVD","2nd CVD"),
+                               split.states = TRUE )
> #
> levels( L1 )
[1] "DM"                               "1st CVD"
[3] "2nd CVD"                          "3rd CVD"
[5] "Dead"                             "Dead(1st CVD)"
[7] "Dead(1st CVD)(2nd CVD)"          "Dead(1st CVD)(2nd CVD)(3rd CVD)"

> levels( Li$lex.Cst )[5:8] <- 
+ levels( Li$lex.Xst )[5:8] <- paste("Dth(,0:3,) ",sep="")
```

29 / 48

```
> boxes( L1, boxpos=list( x=c(rep(seq(15,85,,4),2) ),
+                               y=rep(c(85,20),each=4) ), show.BE=TRUE, scale.R=100 )
```

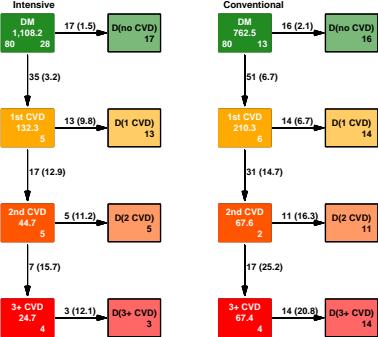


30 / 48

```
> head( st2[,-c(1,4:5,12,13)] )
```

	allocation	sex	doBth	doDM	doBase	doCVD1	doCVD2	doCVD3	doDth
1	Intensive	M	1932.300	1991.093	1993.357	2014.457	NA	NA	NA
2	Intensive	M	1946.813	1982.130	1993.357	2009.231	2009.570	2010.162	NA
3	Conventional	M	1943.377	1982.560	1993.362	2001.639	NA	NA	2001.639
4	Conventional	M	1944.771	1976.791	1993.362	1995.415	1997.461	2003.443	
5	Conventional	M	1935.993	1986.346	1993.162	1993.959	1994.657	1998.047	1998.047
6	Conventional	M	1946.942	1986.404	1993.201	1998.769	NA	NA	NA

25 / 48



31/ 48

An added time scale IV

791	5	1997.162	61.16906	10.815916	4.0000000	3.20328542	0.08333333	2nd	CVD	2
792	5	1997.246	61.25240	10.899250	4.08333333	3.28661875	0.08333333	2nd	CVD	2
793	5	1997.329	61.33573	10.982583	4.16666667	3.36995209	0.08333333	2nd	CVD	2
794	5	1997.412	61.41906	11.065916	4.2500000	3.45328542	0.08333333	2nd	CVD	2
795	5	1997.496	61.50240	11.149250	4.33333333	3.53661875	0.08333333	2nd	CVD	2
796	5	1997.573	61.58573	11.232583	4.41666667	3.61995209	0.08333333	2nd	CVD	2
797	5	1997.662	61.66906	11.315916	4.5000000	3.70328542	0.08333333	2nd	CVD	2
798	5	1997.746	61.75240	11.399250	4.58333333	3.78661875	0.08333333	2nd	CVD	2
799	5	1997.829	61.83573	11.482583	4.66666667	3.86995209	0.08333333	2nd	CVD	2
800	5	1997.912	61.91906	11.565916	4.7500000	3.95328542	0.08333333	2nd	CVD	2
801	5	1997.996	62.00240	11.649250	4.83333333	4.03661875	0.05099247	2nd	CVD	2

36/ 48

An added time scale

We added the timescale tfCVD using new.scale="tfCVD"

```
> subset( L1, lex.id==5 )[,1:9]
```

per	age	dur	tsb	tfCVD	lex.dur	lex.Cst	lex.Xst	lex.id
10	1993.162	57.16906	6.815916	0.0000000	NA	0.7967146	DM 1st CVD	5
11	1993.959	57.96578	7.612631	0.7967146	0.0000000	0.6981520	1st CVD 2nd CVD	5
12	1994.657	58.66393	8.310783	1.4948665	0.698152	3.3894593	2nd CVD Dth(2)	5

32/ 48

An added time scale I

We added the timescale tfCVD using new.scale="tfCVD"

```
> S1 <- splitLexis( L1, breaks=seq(0,25,1/12), time.scale="tsb" )
> subset( S1, lex.id==5 )[,1:9]
```

lex.id	per	age	dur	tsb	tfCVD	lex.dur	lex.Cst	le
741	5	1993.162	57.16906	6.815916	0.0000000	NA	0.08333333	DM
742	5	1993.246	57.25240	6.899250	0.08333333	NA	0.08333333	DM
743	5	1993.329	57.33573	6.982583	0.16666667	NA	0.08333333	DM
744	5	1993.412	57.41906	7.065916	0.2500000	NA	0.08333333	DM
745	5	1993.496	57.50240	7.149250	0.33333333	NA	0.08333333	DM
746	5	1993.579	57.58573	7.232583	0.41666667	NA	0.08333333	DM
747	5	1993.662	57.66906	7.315916	0.5000000	NA	0.08333333	DM
748	5	1993.746	57.75240	7.399250	0.58333333	NA	0.08333333	DM
749	5	1993.829	57.83573	7.482583	0.66666667	NA	0.08333333	DM
750	5	1993.912	57.91906	7.565916	0.7500000	NA	0.04671458	DM 1s
751	5	1993.959	57.96578	7.612631	0.79671458	0.0000000	0.03661875	1st CVD 1s
752	5	1993.996	58.00240	7.649250	0.83333333	0.03661875	0.08333333	1st CVD 1s

33/ 48

Representation of multistate FU: Lexis I

lex.id	per	age	dur	tsb	tfCVD	lex.dur	lex.Cst	lex.Xst	
5	1993.162	57.16906	6.815916	0.0000000	NA	0.08333333	NA	DM	
5	1993.246	57.25240	6.899250	0.08333333	NA	0.08333333	NA	DM	
...	5	1993.912	57.91906	7.565916	0.7500000	NA	0.0467145	DM 1st CVD	
5	1993.959	57.96578	7.612631	0.796714	0.0000000	0.0366187	1st CVD 1st CVD	5	
5	1993.996	58.00240	7.649250	0.833333	0.036618	0.0833333	1st CVD 1st CVD	5	
...	5	1994.579	58.58573	8.232583	1.4166666	0.619952	0.0781998	1st CVD 2nd CVD	5
5	1994.657	58.66393	8.310783	1.4948665	0.69815195	0.0500000	0.1494866	2nd CVD 2nd CVD	5
...	5	1997.912	61.91906	11.565916	4.7500000	3.953285	0.08333333	2nd CVD 2nd CVD	5
5	1997.996	62.00240	11.649250	4.8333333	4.036618	0.0509924	2nd CVD Dth(2)	5	

37/ 48

An added time scale I

We added the timescale tfCVD using new.scale="tfCVD"

```
> S1 <- splitLexis( L1, breaks=seq(0,25,1/12), time.scale="tsb" )
> subset( S1, lex.id==5 )[,1:9]
```

lex.id	per	age	dur	tsb	tfCVD	lex.dur	lex.Cst	le
741	5	1993.162	57.16906	6.815916	0.0000000	NA	0.08333333	DM
742	5	1993.246	57.25240	6.899250	0.08333333	NA	0.08333333	DM
743	5	1993.329	58.33573	7.982583	1.16666667	0.36995209	0.08333333	1st CVD
744	5	1993.412	58.41906	8.065916	1.2500000	0.45328542	0.08333333	1st CVD 1s
745	5	1993.496	58.50240	8.149250	1.33333333	0.53661875	0.08333333	1st CVD 1s
746	5	1993.579	58.58573	8.232583	1.41666667	0.61995209	0.0781998	1st CVD 2s
747	5	1993.662	58.66906	8.315916	1.5000000	0.70328542	0.08333333	2nd CVD 2s
748	5	1993.746	58.75240	8.399250	1.58333333	0.78661875	0.08333333	2nd CVD 2s
749	5	1993.829	58.83573	8.482583	1.66666667	0.86995209	0.08333333	2nd CVD 2s
750	5	1993.912	58.91906	8.565916	1.7500000	0.95328542	0.08333333	2nd CVD 2s
751	5	1993.959	58.96578	8.612631	0.79671458	0.0000000	0.03661875	1st CVD 1s
752	5	1993.996	58.00240	7.649250	0.83333333	0.03661875	0.08333333	1st CVD 1s

34/ 48

Modeling mortality rates in Lexis objects

```
> dlev <- c("D(no CVD)", "D(1 CVD)", "D(2 CVD)", "D(3+ CVD)")
> #
> m0 <- glm( (lex.Xst %in% dlev) ~
+   Ns( tsb, knots=d.kn ) + lex.Cst + allocation,
+   offset = log(lex.dur),
+   family = poisson,
+   data = S1 )
> #
> mi <- update( m0, . ~ . + sex + age ) # the real model
> #
> mi <- update( m1, . ~ . - allocation + allocation:lex.Cst )
> #
> # Test interaction
> anova( mii, mi, test="Chisq" )
```

38/ 48

An added time scale II

```
> clev <- c("1st CVD", "2nd CVD", "3+ CVD")
> #
> c0 <- glm( ( lex.Xst %in% clev) & (lex.Cst!=lex.Xst) ) ~
+   Ns( tsb, knots=d.kn ) + lex.Cst + allocation,
+   offset = log(lex.dur),
+   family = poisson,
+   data = subset( S1, lex.Cst!="3+ CVD" ) )
> #
> ci1 <- update( c0, . ~ . + sex + age )
> #
> ci1 <- update( ci1, . ~ . - allocation + allocation:lex.Cst )
> #
> ci1 <- update( ci1, . ~ . + allocation:tsb )
> #
> # Test interaction & PH
> anova( ci1, ci1, ci1, test="Chisq" )
```

39/ 48

An added time scale III

```
> TM1 <- list( "DM" = list( "D(no CVD)" = m1,
+   "1st CVD" = c1 ),
+   "1st CVD" = list( "D(1 CVD)" = m1,
+   "2nd CVD" = c1 ),
+   "2nd CVD" = list( "D(2 CVD)" = m1,
+   "3+ CVD" = c1 ),
+   "3+ CVD" = list( "D(3+ CVD)" = m1 ) )
```

35/ 48

Simulation: transition object

```
> TM1 <- list( "DM" = list( "D(no CVD)" = m1,
+   "1st CVD" = c1 ),
+   "1st CVD" = list( "D(1 CVD)" = m1,
+   "2nd CVD" = c1 ),
+   "2nd CVD" = list( "D(2 CVD)" = m1,
+   "3+ CVD" = c1 ),
+   "3+ CVD" = list( "D(3+ CVD)" = m1 ) )
```

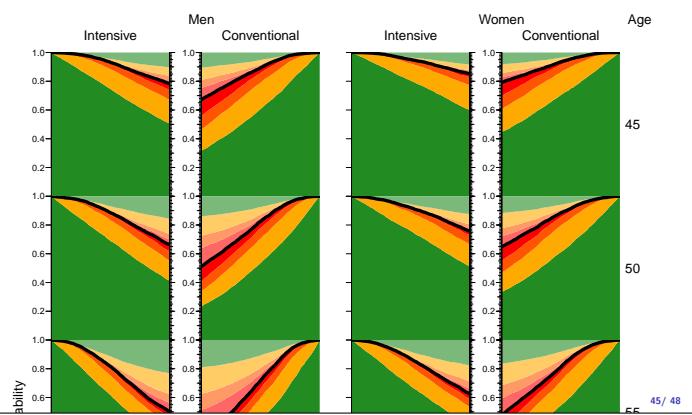
40/ 48

Simulation: baseline (init) object

```
> St2 <- subset( L1, select = c(timeScales(L1),"lex.Cst","lex.dur",
+ "allocation","sex") )
> levels(St2$lex.Cst) <- levels(S1$lex.Cst)
> nr <- nrow(St2)
> St2[1:nr,"tsb"] <- 0
> St2[1:nr,"lex.Cst"] <- levels(St2$lex.Cst)[1]
> St2[1:nr,"allocation"] <- levels(S1$allocation)[1]
> St2 <- rbind( St2, St2 )
> St2[1:nr,"allocation"] <- levels(S1$allocation)[2]
```

St2 is a copy of the original date with allocation="Intensive"
plus a copy of the original date with allocation="Conventional"

41 / 48



Simulating a follow up dataset

```
> sim2 <- simLexis( Tr = TM1,
+                     init = St2,
+                     N = 100,
+                     time pts = seq(0,21,0.1) )
> StC <- nState( subset(sim2,allocation=="Conventional"),
+                  at=seq(0,20,0.1), from=0, time.scale="tsb" )
> StI <- nState( subset(sim2,allocation=="Intensive"),
+                  at=seq(0,20,0.1), from=0, time.scale="tsb" )
> prm <- c(1:4,8:5)
> PC <- pState( StC, perm=prm )
> PI <- pState( StI, perm=prm )
```

42 / 48

History

- ▶ Epi package grew out of "Statistical Practice in Epidemiology with R", annually since 2002 in Tartu Estonia
- ▶ Lexis machinery conceived by Martyn Plummer, IARC
- ▶ Naming originally by David Clayton & Michael Hills, stlexis in Stata, later renamed stsplit
- ▶ David Clayton wrote a lexis function for the Epi package. Obsolete now.

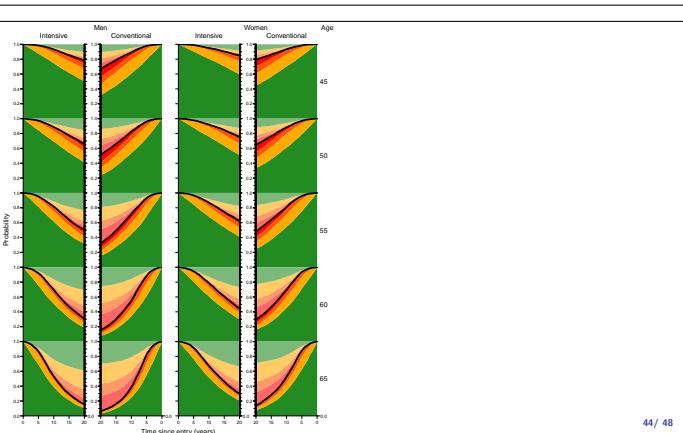
46 / 48



Summary

- ▶ Proper representation of multistate data essential: States, transitions, risk time
- ▶ Readable modeling code
- ▶ Calculation of state probabilities requires a simulation in any realistic situation
- ▶ Examples of use in:
<http://bendixcarstensen.com/AdvCoh/Lexis-ex/>

Thanks for your attention



44 / 48

48 / 48