Practice in analysis of multistate models using

Epi::Lexis in 😱



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Chapter 1

Introduction

There are two practicals in this document. This first one, "Renal complications: Time-dependent variables and multiple states" is the one we shall be using, the other is included for self-study if you wish so. Both exercises also has a solution-version in following chapter.

1.1 Computing prerequisites

The practicals assume that you have an up to date version of R (3.3.1) as well as the last version of the Epi package (2.5), the following should work and give you the relevant information:

```
> install.packages( "Epi" )
> library( Epi )
> sessionInfo()
```

Also, you will need to access a dataset from the website so you might want to download the file it is http://bendixcarstensen.com/AdvCoh/courses/Frias-2016/renal.Rda

In the tutorial I shall assume that you are familiar with the following commands in R:

- glm, including the offset= argument
- update for models
- predict, and the wrapper ci.pred from the Epi package.

1.2 Statistical prerequisites

I will assume that you are familiar with the usual likelihood machinery and the theory of generalized linear models.

And of course the basic probability theory underlying calculation of demographic rates and probabilities derived from these.

Chapter 2

Exercises

2.1 Renal complications: Time-dependent variables and multiple states

The following practical exercise is based on the data from paper:

P Hovind, L Tarnow, P Rossing, B Carstensen, and HH Parving: Improved survival in patients obtaining remission of nephrotic range albuminuria in diabetic nephropathy. *Kidney Int*, **66**(3):1180–1186, Sept 2004.

You can find a .pdf-version of the paper here:

http://BendixCarstensen.com/~bxc/AdvCoh/papers/Hovind.2004.pdf

2.1.1 The renal failure dataset

The dataset renal.dta contains data on follow up of 125 patients from Steno Diabetes Center. They enter the study when they are diagnosed with nephrotic range albuminuria (NRA). This is a condition where the levels of albumin in the urine is exceeds a certain level as a sign of kidney disease. The levels may however drop as a consequence of treatment, this is called remission. Patients exit the study at death or kidney failure (dialysis or transplant).

Table 2.1: Variables in renal.dta.

id	Patient id
sex	M / F
dob	Date of birth
doe	Date of entry into the study (2.5 years after NRA)
dor	Date of remission. Missing if no remission has occurred
dox	Date of exit from study
event	Exit status: 0: censored, 1: death, 2: end stage renal disease,
	ESRD (kidney failure) and 3: Kidney transplant.

1. The dataset is available at the course website as renal.Rda:

```
library( Epi ) ; clear()
load( url("http://BendixCarstensen.com/AdvCoh/courses/Frias-2016/data/renal.Rda") )
# load( "renal.Rda" )
str( renal )
head( renal )
```

Here we shall only be interested in the combined event 1, 2 or 3.

2. Use the Lexis function to declare the data as survival data with age, calendar time and time since entry into the study as timescales. Note that any coding of event > 0will be labeled "ESRD", i.e. renal death (death of kidney (transplant or dialysis), or person).

Note that you must make sure that the "alive" state (here NRA) is the first, as Lexis assumes that everyone starts in this state (unless of course entry.status is specified).

```
Lr <- Lexis( entry = list( per=doe,</pre>
                             age=doe-dob,
                             tfi=0),
              exit = list( per=dox ),
       exit.status = factor( event>0, labels=c("NRA", "ESRD") ),
               data = renal )
str( Lr )
summary( Lr )
```

3. Visualize the data in a Lexis-diagram, using the plot method for Lexis objects. What do you see?

```
plot( Lr, col="black", lwd=3 )
```

4. (Optional, not crucial to the rest of the exercise. Now try to produce a slightly more fancy Lexis diagram. Note that we have a x-axis of 40 years, and a y-axis of 80 years, so when specifying the output file adjust the total width of the plot so that the use mai to specify the margins of the plot leaves a plotting area twice as high as wide. You will want to consult the maning of the argument mai to the function par.

```
# pdf( "lexis-fancy.pdf", height=80/5+1, width=40/5+1 )
# x11( height=80/5+1, width=40/5+1 )
par( mai=c(3,3,1,1)/4, mgp=c(3,1,0)/1.6 )
plot( Lr, 1:2, col=c("blue", "red")[Lr$sex], lwd=3, grid=0:20*5,
      xlab="Calendar time", ylab="Age",
      xlim=c(1970,2010), ylim=c(0,80), xaxs="i", yaxs="i", las=1)
# dev.off()
```

5. Make a Cox-regression analysis with the variables sex and age at entry into the study, using time since entry to the study as time scale.

Give the hazard ratio between males and females and between two persons who differ 10 years in age at entry. Give the 95% confidence intervals for this as well.

6. The main focus of the paper was to assess whether occurrence of remission (return to a lower level of albumin excretion, an indication of kidney recovery) influences mortality.

"Remission" is a time-dependent variable which is initially 0, but takes the value 1 when remission occurs. In order to handle this, each person who see a remission must have two records:

- One record for the time before remission, where entry is doe, exit is dor, remission is 0, and event is 0.
- One record for the time after remission, where entry is dor, exit is dox, remission is 1, and event is 0 or 1 according to whether the person had an event at dox.

This is accomplished using the cutLexis function on the Lexis object. You must declare the "NRA" state as a precursor state, i.e. a state that is *less* severe than "Rem" in the sense that a person who see a remission will stay in the "Rem" state unless he goes to the "ESRD" state.

List records for a few select persons from Lr and from Lc to see how the cut has worked.

7. Show how the states are connected and the number of transitions between them by using boxes. This is an interactive command that requires you to click in the graph window:

```
boxes( Lc )
```

Alternatively you can let R try to place the boxes for you, and even compute rates (in this case in units of events per 100 PY):

```
boxes( Lc, boxpos=TRUE, scale.R=100, show.BE=TRUE )
```

How many transitions are there from remission to death?

8. (Optional: Not relevant for the remainder of the exercise.) Now make a Lexis diagram where different colouring is used for different segments of the follow-up—you should be able to count the 8 transitions from "Rem" to "ESRD".

9. Make a Cox-regression of mortality (i.e. endpoint "ESRD") with sex, age at entry and remission as explanatory variables, and using time since entry as timescale.

Remember to include lex.Cst as time-dependent variable, and to indicate that each recort represbts follow-up from tfi to tfi+lex.dur. Note the use of the Lexis variables lex.dur (risk time), lex.Xst (exit status) and lex.Cst (current status).

```
m1 <- coxph( Surv( tfi, tfi+lex.dur, lex.Xst=="ESRD" ) ~ sex + I((doe-dob-50)/10) + (lex.Cst=="Rem"), data=Lc ) summary( m1 )
```

- 10. What is the relation between the rate of ESRD between persons in remission and persons not?
- 11. What is the assumption about the two rates of remission? Refer to the figure with the three boxes you just made. (??).

2.1.2 Splitting the follow-up time

In order to explore the effect of remission on the rate of ESRD, we will split the data further into small pieces of follow-up. To this end we use the function **splitLexis**. The rates can then be modeled using a Poisson-model, and the shape of the underlying *rates* be explored. Furthermore, we can allow effects of both time since NRA and current age. To this end we will use splines, so we need the splines package, too.

12. First, split the follow-up time every month after entry, and make sure that the number of events and risk time is the same as before (use summary):

```
sLc <- splitLexis( Lc, "tfi", breaks=seq(0,30,1/12) )
summary( Lc )
summary(sLc )</pre>
```

13. Now try to fit the Poisson-model corresponding to the Cox-model we fitted previously. The function ns() produces a model matrix corresponding to a piecewise cubic function, modeling the baseline hazard explicitly (think of the ns terms as the baseline hazard that is not visible in the Cox-model).

The outcome is 1 or 0 according to whether an event occurred or not, but sine a Poisson variate by definition is numerical, R will automatically coerce (change) a logical value to numeric; FALSE as 0 and TRUE as 1, so we can conveniently write:

The ns function places knots at the quantiles of the variable, which may not be the most logica as the information is contained in the events, so the natural placement of knots would be at the quantiles of the event times. The Ns function in the Epi package automatically takes the smallest and the largest of the knots as boundary knots — the nuber of parameters is one less then the number of knots, so we use 5 knots:

```
t.kn \leftarrow with(\ subset(\ sLc,\ lex.Xst="ESRD"), \\ quantile(\ tfi+lex.dur,\ 0:4/5\ )) mp \leftarrow glm(\ lex.Xst="ESRD" \sim Ns(\ tfi,\ knots=t.kn\ ) + \\ sex + I((doe-dob-40)/10) + (lex.Cst=="Rem"), \\ offset = log(lex.dur), \\ family = poisson, \\ data = sLc\ ) summary(\ mp\ )
```

14. You can extract the parameters from the models using ci.lin or ci.exp try:

```
ci.lin( mp )
ci.exp( mp )
ci.exp( mp, subset=c("sex","dob","Cst"), pval=TRUE )
```

Compare with the estimates from the Cox-model. Use:

```
ci.exp( m1 )
ci.exp( mp, subset=c("sex","dob","Cst") )
ci.exp( mp, subset=c("sex","dob","Cst") ) / ci.exp( m1 )
```

What do you conclude about the models?

15. You can visualize the spline term using termplot, try:

```
termplot( mp, terms=1 )
```

... which is not a terribly informative plot

16. termplot does not give you the absolute level of the underlying rates because it bypasses the intercept. If you explicitly include the intercept in the baseline split you can use Termplot from the Epi package to get estimates on the rate scale for a reference person (in units of events per 100 years):

```
mP <- glm( lex.Xst=="ESRD" ~ -1 + Ns( tfi, knots=t.kn, intercept=TRUE ) + sex + I((doe-dob-40)/10) + (lex.Cst=="Rem"), offset = log(lex.dur/100), family = poisson, data = sLc )

Termplot( mP, terms=1 )
```

How would you describe this rate function in plain words? And what is the scale of the y-axis.

Annotate the axes of the plot accordingly — consult the help page of Termplot.

17. Apart from the baseline timescale, time since NRA, time since remission might be of interest in describing the mortality rate. However this is only relevant for persons who actually have a remission, so start by checking how many events there are in this group:

```
summary( sLc )
```

How many go in remission, and how many deaths are in this group?

18. With this rather limited number of events we can certainly not expect to be able to model anything more complicated than a linear trend with time since remission. Two parameters on 8 events is actually pretty far-fetched.

The variable we want to have in the model is current date (per) minus date of remission (dor): per-dor), but *only* positive values of it. This can be fixed by using pmax(), but we must also deal with all those who have missing values, so we use the construct:

```
pmax( per-dor, 0, na.rm=TRUE )
```

Make sure that you understand what goes on here.

19. We can now expand the model with this variable:

```
sLc \leftarrow transform(\ sLc,\ tfr = pmax(\ (per-dor)/10,\ 0,\ na.rm=TRUE\ )\ ) mPx \leftarrow glm(\ lex.Xst="ESRD"\ ^-1 + Ns(\ tfi,\ knots=t.kn,\ intercept=TRUE\ ) + sex + I((age-tfi-40)/10) + (lex.Cst="Rem") + tfr, offset = log(lex.dur/100), family = poisson, data = sLc\ ) round(\ ci.exp(\ mPx\ ),\ 3\ ) Termplot(\ mPx,\ terms=1\ )
```

- 20. Is the effect significant? Can a substantial effect of time since remission be ruled out?
- 21. What is the test of this parameter traditionally called? What is the null and what is the alternative of this test?

2.1.3 Prediction in a multistate model

This part of the practical is about making proper statements about the survival and the disease probabilities. But in order to do this we must know not only how the occurrence of remission influences the rate of death/ESRD, but we must also model the occurrence rate of remission itself.

The following exercise will be quite similar to the example in the help file for **simLexis** (which you should read now!).

22. The rates of ESRD were modelled by a Poisson model with effects of age and time since NRA — in the model mp. But in the modelling of the remission rates transition from "NRA" to "Rem", the number of events is rather small, so we restrict the variables in this model to only time since NRA and sex. Also remember, only the records that relate to the "NRA" state can be used:

23. If we want to predict the probability of being in each of the three states using these estimated rates, we can either do analytical calculations of the probabilities from the estimated rates, or we can *simulate* the life course through a model using the estimated rates. That will give a simulated cohort (in the form of a Lexis object), and we can then just count the number of persons in each state at each of a set of time points.

This is accomplished using the function simLexis. The input to this is the initial status of the persons whose life-course we shall simulate, and the transition rates in suitable form:

• Suppose we want predictions for men aged 50 at NRA. The input is in the form of a Lexis object (where lex.dur and lex.Xst will be ignored). Note that in order to carry over the time.scales and the time.since attributes, we construct the input object using subset to select columns, and NULL to select rows (see the example in the help file for simLexis):

```
inL <- subset( sLc, select=1:11 )[NULL,]
str( inL )
timeScales(inL)
inL[1,"lex.id"] <- 1
inL[1,"per"] <- 2000
inL[1,"age"] <- 50
inL[1,"tfi"] <- 0
inL[1,"tfi"] <- 0
inL[1,"lex.Cst"] <- "NRA"
inL[1,"lex.Xst"] <- NA
inL[1,"lex.dur"] <- NA
inL[1,"doe"] <- 2000
inL[1,"dob"] <- 1950
inL</pre>
```

• The other input for the simulation is the transitions, which is a list with an element for each transient state (that is "NRA" and "Rem"), each of which is again a list with names equal to the states that can be reached from the transient state. The content of the list will be glm objects, in this case the models we just fitted, describing the transition rates:

With this as input we can now generate a cohort, using N=10 to simulate life course of 10 persons (with identical starting values):

```
( iL <- simLexis( Tr, inL, N=10 ) )
summary( iL )</pre>
```

24. Now generate the life course of 10,000 persons, and look at the summary. The system.time command is just to tell you how long it took, you may want to start with 1000 just to see how long that takes.

```
system.time(
sM <- simLexis( Tr, inL, N=10000 ) )
summary( sM )</pre>
```

Why are there so many ESRD-events in the resulting data set?

25. Now we want to count how many persons are present in each state at each time for the first 10 years after entry (which is at age 50). This can be done by using nState:

```
nSt \leftarrow nState(sM, at=seq(0,10,0.1), from=50, time.scale="age") head(<math>nSt)
```

26. Once we have the counts of persons in each state at the designated time points, we compute the cumulative fraction over the states, arranged in order given by perm:

```
pp <- pState( nSt, perm=1:3 )
head( pp )
tail( pp )</pre>
```

27. Try to plot the cumulative probabilities using the plot method for pState objects:

```
plot(pp)
```

28. A quantity of particular interest would be how many patients actually get a remission. This is not deductible from the plot just shown, because those who get ESRD are not subdivided according to whether they have a remission prior to ESRD.

The simplest way to doctor that is to modify the simulated object (sM in the above notation), so that those exiting to "ESRD" from "Rem" are counted in a separate state. We must also change the formal set of levels of lex.Cst:

29. Having done this, try to compute the number of persons in each of the 4 states, and the cumulative proportions to be plotted:

```
xSt <- nState( xM, at=seq(0,10,0.1), from=50, time.scale="age" )
xp <- pState( xSt, perm=1:4 )
head( xp )
plot( xp, col=rev(c("pink","limegreen","forestgreen","red")), xlab="Age" )
lines( as.numeric(rownames(xp)), xp[,"Rem"], lwd=4 )</pre>
```

What is the probability that a 50-year old man with NRA sees a remission from NRA during the next 10 yezrs?

- 30. Make the same calculations for a 60-year old woman.
- 31. Normally you would know that a split of the absorbing "ESRD" state according to the preceding state and so define this in the cutLexis function, using split.states. At the same time it is also possible to define a new timescale using new.scale, defined as time since entry to the new state:

```
Lc <- cutLexis( Lr, cut = Lr$dor, # where to cut follow up
               timescale = "per", # the timescale that "dor" refers to
              new.state = "Rem",
                                   # name of the new state
       precursor.states = "NRA", # which states are less severe
              new.scale = "tfr", # define a new timescale as time since Rem
           split.states = TRUE ) # subdivide non-precursor states
str(Lc)
# source("/home/bendix/stat/R/lib.src/Epi/pkg/R/summary.Lexis.r")
# summary( Lc, S=T, scale=100 )
summary( Lc )
boxes(Lc, boxpos=list(x=c(20,80,20,80), y=c(80,80,20,20)),
           scale.R=100, show.BE=TRUE )
sLc <- splitLexis( Lc, "tfi", breaks=seq(0,30,1/12) )</pre>
summary( Lc )
summary( sLc )
head( subset( sLc, lex.id==2 )[,1:8], 8 )
tail( subset( sLc, lex.id==2 )[,1:8], 3 )
( fl <- levels(Lc)[3:4] )
mp \leftarrow glm(lex.Xst \%in\% fl ~ns(tfi, df=4) +
                              sex + I((age-tfi-40)/10) + (lex.Cst=="Rem"),
           offset = log(lex.dur/100),
           family = poisson,
             data = sLc )
# the timescale tfr must be given some value for time before Rem
sLc$tfr <- pmax( 0, sLc$tfr, na.rm=TRUE )</pre>
head( subset( sLc, lex.id==2 )[,1:8], 8 )
mr \leftarrow glm(lex.Xst=="Rem" ~ ns(tfi, df=4) + sex,
           offset = log(lex.dur),
           family = poisson,
              data = subset( sLc, lex.Cst=="NRA" ) )
ci.exp( mr, pval=TRUE )
inL <- subset( sLc, select=1:10 )[NULL,]</pre>
str( inL )
timeScales(inL)
inL[1,"lex.id"] <- 1
inL[1, "per"] <- 2000
inL[1, "age"] <- 50
inL[1, "tfi"] <- 0
```

```
inL[1,"lex.Cst"] <- "NRA"
inL[1,"lex.Xst"] <- NA</pre>
inL[1,"lex.dur"] <- NA</pre>
inL[1, "sex"] <- "M"
inL
Tr \leftarrow list("NRA" = list("Rem" = mr,
                            "ESRD" = mp ),
             "Rem" = list( "ESRD(Rem)" = mp ) )
( iL <- simLexis( Tr, inL, N=10 ) )
summary( iL )
system.time(
sM <- simLexis( Tr, inL, N=10000, t.range=25, n.int=251 ) )</pre>
summary( sM )
nSt <- nState( sM, at=seq(0,24,0.1), from=50, time.scale="age" )</pre>
head( nSt )
pp \leftarrow pState(nSt, perm=c(1,2,4,3))
head( pp )
tail(pp)
plot( pp )
# Two colors and the corresponding pale ones for the dead states
clr <- c("limegreen", "orange")</pre>
col2rgb(clr)
c14 \leftarrow cbind(col2rgb(clr), col2rgb(clr)/2+255/2)[, c(1,2,4,3)]
c14 \leftarrow rgb(t(c14), max=255)
# Nicer plot
plot( pp, col=cl4, xlab="Age" )
lines( as.numeric(rownames(pp)), pp[,2], lwd=2 )
```

2.2 Time-splitting, time-scales and SMR: Diabetes in Denmark

This exercise is using data from the National Danish Diabetes register. There is a random sample of 10,000 records from this in the Epi package. Actually there are two data sets, we shall use the one with only cases of diabetes diagnosed after 1995, see the help page for DM1ate.

This is of interest because it is only for these where the data of diagnosis is certain, and hence for whom we can compute the duration of diabetes during follow-up.

The exercise is about assessing how mortality depends age, calendar time and duration of diabetes. And how to understand and compute SMR, and assess how it depends on these factors as well.

1. First load the data and take a look at the data:

```
> library( Epi )
> data( DMlate )
> str( DMlate )
```

You can get a more detailed explanation of the data by referring to the help page:

```
> ?DMlate
```

2. Set up the dataset as a Lexis object with age, calendar time and duration of diabetes as timescales, and date of death as event. Make sure that you know what each of the arguments to Lexis mean:

Take a look at the first few lines of the resulting dataset using head().

- 3. Get an overall overview of the mortality by using stat.table to tabulate no. deaths, person-years and the crude mortality rate by sex.
- 4. If we want to assess how mortality depends on age, calendar time and duration, we should split the follow-up along all three time scales. In practice it is sufficient to split it along one of the time-scales and then just use the value of each of the time-scales at the left endpoint of the intervals.

Use splitLexis to split the follow-up along the age-axis:

```
> SL <- splitLexis( LL, breaks=seq(0,125,1/2), time.scale="A" )
> summary( SL )
```

How many records are now in the dataset? How many person-years? Compare to the original Lexis-dataset.

5. Now estimate an age-specific mortality curve for men and women separately, using natural splines:

Make sure you understand all the components on this modeling statement.

6. Now try to get the estimated rates by using the wrapper function ci.pred that computes predicted rates and confidence limits for these.

Note that lex.dur is a covariate in the context of prediction; by putting this to 1000 in the prediction dataset we get the rates in units of deaths per 1000 PY:

```
> nd <- data.frame( A = seq(10,90,0.5),
+ lex.dur = 1000)
> p.m <- ci.pred( r.m, newdata = nd )
> str( p.m )
```

7. Plot the predicted rates for men and women together - using for example matplot.

Period and duration effects

8. We now want to model the mortality rates among diabetes patients also including current date and duration of diabetes. However, we shall not just use the positioning of knots for the splines as provided by ns, because this is based on the allocating knots so that the number of observations in the dataset is the same between knots. The information in a follow-up study is in the number of events, so it would be better to allocate knots so that number of events were the same between knots.

We take the 5th and 95th percentile of deaths as the boundary knots for age (A) and calendar time (P), but for duration (dur) where we actually have follow-up from time 0 on the timescale, we use 0 as the first knot.

Therefore, find points (knots) so that the number of events is the same between each pair. Try this:

Take a look at where these points are and make a similar construction for calendar time (P) and diabetes duration (dur) — remember to add 0 as a knot for the latter.

9. With knots for age, period and duration we can now model mortality rates (separately for men and women), as functions of age, calendar time and duration of diabetes. To this end you will need the function Ns from the Epi package (look that up) to specify a model very simply

10. How do these models fit relative to the models with only age as a descriptor of the rates?

(Hint: Use the anova-function with the argument ${\tt test="Chisq"}$ to compare the models.

11. If we want to see the shape of the three effects we can use the type="terms" facility in the predict.glm that makes predictions separately for each term in the model. But this does not include the intercept, so if we want prediction of terms that add up to the total predicted value we must explictly include the intercept in one of the terms; age, say, thereby making age the term with a rate-dimension and interpretable as age-specific rates.

This requires that we select reference points for the other terms, period and duration.

This is done by using the intercept and ref arguments to Ns:

Check that it actually is the same model, for example by using the deviances from the two models fitted.

12. Once this is done we can use Termplot, which is a wrapper for termplot. Termplot gives plots on the rate / resp RR scale, so that we can actually make sens of the plots. Now make a plot of the three effects in the model:

```
> Termplot( mx )
```

What is the interpretation of the three terms in the model?

13. The model we fitted has three time-scales: current age, current date and current duration of diabetes, so the effects that we report are not immediately interpretable, as they are (as in any kind of multiple regressions) to be interpreted as "all else equal" which they are not, as the three time scales advance simultaneously at the same pace.

The reporting would therefore more naturally be *only* on the mortality scale, but showing the mortality for persons diagnosed in different ages, using separate displays for separate years of diagnosis.

This is most easily done using the ci.pred function with the newdata= argument. So a person diagnosed in age 50 in 1995 will have a mortality measured in cases per 1000 PY as:

Now take a look at the result from the ci.pred statement and construct prediction of mortality for men and women diagnosed in a range of ages, say 50, 60, 70, and plot these together in the same graph.

2.2.1 SMR

The SMR is the Standardized Mortality Ratio, which is the mortality rate-ratio between the diabetes patients and the general population. In real studies we would subtract the deaths and the person-years among the diabetes patients from those of the general population, but since we do not have access to these, we make the comparison to the general population at large, *i.e.* also including the diabetes patients.

There are two ways to make the comparison to the population mortality; one is to amend the diabetes patient dataset with the population mortality dataset, the other (classical) one is to include the population mortality rates as a fixed variable in the calculations. The latter requires that each analytical unit in the diabetes patient dataset is amended with a variable with the population mortality rate for the corresponding sex, age and calendar time.

This can be achieved in two ways: Either we just use the current split of follow-up time and allocate the population mortality rates for some suitably chosen (mid-)point of the follow-up in each, or we make a second split by date, so that follow-up in the diabetes patients is in the same classification of age and data as the population mortality table.

14. We will use the former approach, that is in the diabetes dataset to include as an extra variable the population mortality as available from the data set M.dk.

First create the variables in the diabetes dataset that we need for matching with the population mortality data, that is age, date and sex at the midpoint of each of the intervals (or rater at a point 3 months after the left endpoint of the interval — recall we split the follow-up in 6 month intervals).

We need to have variables of the same type when we merge, so we must transform the sex variable in M.dk to a factor, and must for each follow-up interval in the SL data have an age and a period variable that can be used in merging with the population data.

Then match the rates from M.dk into SL — sex, Am and Pm are the common variables, and therefore the match is on these variables:

```
> SLr <- merge( SL, M.dk[,c("sex","Am","Pm","rate")] )
> dim( SL )
> dim( SLr )
```

This merge only takes rows that have information from both datasets, hence the slightly fewer rows in SLr than in SL.

- 15. Compute the expected number of deaths as the person-time multiplied by the corresponding population rate, and put it in a new variable. Use stat.table to make a table of observed, expected and the ratio (SMR) by age (suitably grouped) and sex.
- 16. Then model the SMR using age and date of diagnosis and diabetes duration as explanatory variables, including the log-expected-number instead of the log-person-years as offset, using separate models for men and women. Remember to exclude those units where no deaths in the population occur (that is where the rate is 0).

Plot the estimates as you did before for the rates, using Termplot. What do the extracted effects represent now?

- 17. Is there any difference between SMR for males and females?
- 18. Plot the predicted SMR as you did the predicted rates for persons aged 50, 60 and 70 at diagnosis.
- 19. Try to simplify the model to one with a simple linear effect of date of diagnosis, and using only knots at 0,1,and 2 years for duration, giving an estimate of the change in SMR as duration increases beyond 2 years.
- 20. What are the estimated annual change in SMR by date of diagnosis and by duration after 2 years?

Chapter 3

Solutions

3.1 Renal complications: Time-dependent variables and multiple states

The following practical exercise is based on the data from paper:

P Hovind, L Tarnow, P Rossing, B Carstensen, and HH Parving: Improved survival in patients obtaining remission of nephrotic range albuminuria in diabetic nephropathy. *Kidney Int*, **66**(3):1180–1186, Sept 2004.

You can find a .pdf-version of the paper here:

http://BendixCarstensen.com/~bxc/AdvCoh/papers/Hovind.2004.pdf

3.1.1 The renal failure dataset

The dataset renal.dta contains data on follow up of 125 patients from Steno Diabetes Center. They enter the study when they are diagnosed with nephrotic range albuminuria (NRA). This is a condition where the levels of albumin in the urine is exceeds a certain level as a sign of kidney disease. The levels may however drop as a consequence of treatment, this is called remission. Patients exit the study at death or kidney failure (dialysis or transplant).

Table 3.1: Variables in renal.dta.

id	Patient id
sex	$\mathrm{M}\ /\ \mathrm{F}$
dob	Date of birth
doe	Date of entry into the study (2.5 years after NRA)
dor	Date of remission. Missing if no remission has occurred
dox	Date of exit from study
event	Exit status: 0: censored, 1: death, 2: end stage renal disease,
	ESRD (kidney failure) and 3: Kidney transplant

1. The dataset is available at the course website as renal.Rda:

```
library( Epi ) ; clear()
 load( url("http://BendixCarstensen.com/AdvCoh/courses/Frias-2016/renal.Rda") )
 # load( "renal.Rda" )
 str(renal)
'data.frame':
                     125 obs. of 7 variables:
       : num 17 26 27 33 42 46 47 55 62 64 ...
 $ sex : Factor w/ 2 levels "M", "F": 1 2 2 1 2 2 1 1 2 1 ...
              1968 1959 1962 1951 1961 ...
       : num
 $ doe : num
               1996 1990 1988 1995 1988 ...
 $ dor : num
               NA 1990 NA 1996 1997 ...
 $ dox : num 1997 1996 1993 2004 2004
               2 1 3 0 0 2 1 0 2 0 ...
 $ event: num
 head( renal )
  id sex
              dob
                       doe
                                dor
                                         dox event
       M 1967.944 1996.013
1 17
                                 NA 1997.094
2 26
       F 1959.306 1989.535 1989.814 1996.136
                                                 1
3 27
       F 1962.014 1987.846
                                 NA 1993.239
4 33
       M 1950.747 1995.243 1995.717 2003.993
                                                 0
       F 1961.296 1987.884 1996.650 2003.955
                                                 0
5 42
6 46
      F 1952.374 1983.419
                                 NA 1991.484
```

Here we shall only be interested in the combined event 1, 2 or 3.

2. Use the Lexis function to declare the data as survival data with age, calendar time and time since entry into the study as timescales. Note that any coding of event > 0 will be labeled "ESRD", i.e. renal death (death of kidney (transplant or dialysis), or person).

Note that you must make sure that the "alive" state (here NRA) is the first, as Lexis assumes that everyone starts in this state (unless of course entry.status is specified).

```
Lr <- Lexis( entry = list( per=doe,</pre>
                            age=doe-dob,
                            tfi=0 ),
               exit = list( per=dox );
        exit.status = factor( event>0, labels=c("NRA", "ESRD") ),
               data = renal )
NOTE: entry.status has been set to "NRA" for all.
 str(Lr)
Classes 'Lexis' and 'data.frame':
                                         125 obs. of 14 variables:
         : num 1996 1990 1988 1995 1988 ...
 $ age
          : num 28.1 30.2 25.8 44.5 26.6 ...
          : num 0000000000...
 $ lex.dur: num 1.08 6.6 5.39 8.75 16.07 ...
 $ lex.Cst: Factor w/ 2 levels "NRA","ESRD": 1 1 1 1 1 1 1 1 1 1 1 ...
 $ lex.Xst: Factor w/ 2 levels "NRA","ESRD": 2 2 2 1 1 2 2 1 2 1 ...
 $ lex.id : int
                 1 2 3 4 5 6 7 8 9 10 ...
 $ id
                 17 26 27 33 42 46 47 55 62 64 ...
          : num
          : Factor w/ 2 levels "M", "F": 1 2 2 1 2 2 1 1 2 1 ...
 $ sex
 $ dob
          : num 1968 1959 1962 1951 1961 ...
 $ doe
          : num 1996 1990 1988 1995 1988 ...
```

```
$ dor : num NA 1990 NA 1996 1997 ...
$ dox : num 1997 1996 1993 2004 2004 ...
$ event : num 2 1 3 0 0 2 1 0 2 0 ...
- attr(*, "time.scales")= chr "per" "age" "tfi"
- attr(*, "time.since")= chr "" "" ""
- attr(*, "breaks")=List of 3
    ..$ per: NULL
    ..$ age: NULL
    ..$ tfi: NULL

Summary( Lr )

Transitions:
    To
From NRA ESRD Records: Events: Risk time: Persons:
    NRA 48 77 125 77 1084.67 125
```

3. Visualize the data in a Lexis-diagram, using the plot method for Lexis objects. What do you see?

```
plot( Lr, col="black", lwd=3 )
```

4. (Optional, not crucial to the rest of the exercise. Now try to produce a slightly more fancy Lexis diagram. Note that we have a x-axis of 40 years, and a y-axis of 80 years, so when specifying the output file adjust the total width of the plot so that the use mai to specify the margins of the plot leaves a plotting area twice as high as wide. You will want to consult the maning of the argument mai to the function par.

5. Make a Cox-regression analysis with the variables sex and age at entry into the study, using time since entry to the study as time scale.

Give the hazard ratio between males and females and between two persons who differ 10 years in age at entry. Give the 95% confidence intervals for this as well.

```
sexF
         -0.1817
                   0.8338
                            0.2727 - 0.666
                                            0.505
         exp(coef) exp(-coef) lower .95 upper .95
I(age/10)
         1.7357
                    0.5761
                             1.3186
sexF
            0.8338
                      1.1993
                                0.4886
                                          1.423
Concordance= 0.612 (se = 0.036)
Rsquare= 0.121 (max possible= 0.994)
                                      p=0.0003237
Likelihood ratio test= 16.07 on 2 df,
Wald test = 16.38 on 2 df,
                                      p=0.0002774
Score (logrank) test = 16.77 on 2 df,
                                      p=0.0002282
```

 The main focus of the paper was to assess whether occurrence of remission (return to a lower level of albumin excretion, an indication of kidney recovery) influences mortality.

"Remission" is a time-dependent variable which is initially 0, but takes the value 1 when remission occurs. In order to handle this, each person who see a remission must have two records:

- One record for the time before remission, where entry is doe, exit is dor, remission is 0, and event is 0.
- One record for the time after remission, where entry is dor, exit is dox, remission is 1, and event is 0 or 1 according to whether the person had an event at dox.

This is accomplished using the cutLexis function on the Lexis object. You must declare the "NRA" state as a precursor state, i.e. a state that is *less* severe than "Rem" in the sense that a person who see a remission will stay in the "Rem" state unless he goes to the "ESRD" state.

```
Lc <- cutLexis( Lr, cut = Lr$dor, # where to cut follow up
          timescale = "per", # the timescale that "dor" refers to
    new.state = "Rem", # name of the new state
precursor.states = "NRA" ) # which states are less severe
 summary( Lc )
Transitions:
      To
From NRA Rem ESRD Records: Events: Risk time:
  NRA 24 29
                    69
                                122
                                            98
                                                      824.77
  Rem
         0
              24
                    8
                                32
                                             8
                                                      259.90
                                                                        32
  Sum 24 53
                    77
                                154
                                            106
                                                     1084.67
                                                                       125
```

List records for a few select persons from Lr and from Lc to see how the cut has worked.

7. Show how the states are connected and the number of transitions between them by using boxes. This is an interactive command that requires you to click in the graph window:

```
boxes (Lc)
```

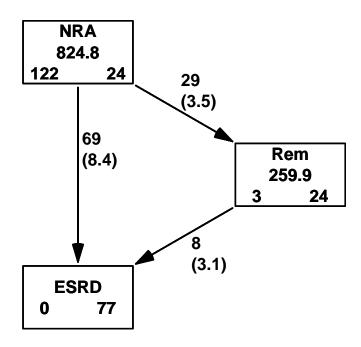


Figure 3.1

Alternatively you can let R try to place the boxes for you, and even compute rates (in this case in units of events per 100 PY):

```
boxes( Lc, boxpos=TRUE, scale.R=100, show.BE=TRUE )
```

How many transitions are there from remission to death?

8. (Optional: Not relevant for the remainder of the exercise.) Now make a Lexis diagram where different colouring is used for different segments of the follow-up—you should be able to count the 8 transitions from "Rem" to "ESRD".

9. Make a Cox-regression of mortality (i.e. endpoint "ESRD") with sex, age at entry and remission as explanatory variables, and using time since entry as timescale.

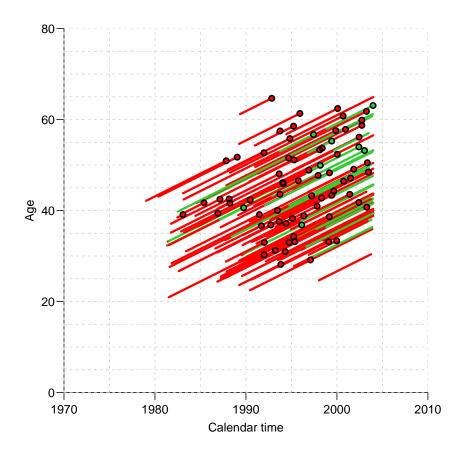


Figure 3.2: Lexis diagram for the split data, where time after remission is shown in green.

Remember to include lex.Cst as time-dependent variable, and to indicate that each recort represbts follow-up from tfi to tfi+lex.dur. Note the use of the Lexis variables lex.dur (risk time), lex.Xst (exit status) and lex.Cst (current status).

```
m1 <- coxph( Surv( tfi, tfi+lex.dur, lex.Xst=="ESRD" ) ~</pre>
             sex + I((doe-dob-50)/10) + (lex.Cst=="Rem"), data=Lc)
summary( m1 )
Call:
coxph(formula = Surv(tfi, tfi + lex.dur, lex.Xst == "ESRD") ~
   sex + I((doe - dob - 50)/10) + (lex.Cst == "Rem"), data = Lc)
 n= 154, number of events= 77
                         coef exp(coef) se(coef)
                                                    z \Pr(>|z|)
sexF
                     -0.05534
                               I((doe - dob - 50)/10) 0.52190
                               1.68522
                                       0.13655 3.822 0.000132
lex.Cst == "Rem"TRUE
                     -1.26241
                               exp(coef) exp(-coef) lower .95 upper .95
sexF
                        0.9462
                                  1.0569
                                           0.5519
                                                     1.6220
I((doe - dob - 50)/10)
                        1.6852
                                  0.5934
                                           1.2895
                                                     2.2024
lex.Cst == "Rem"TRUE
                        0.2830
                                  3.5339
                                           0.1331
                                                     0.6016
Concordance= 0.664 (se = 0.036)
Rsquare= 0.179 (max possible= 0.984)
```

```
Likelihood ratio test= 30.31 on 3 df, p=1.189e-06 Wald test = 27.07 on 3 df, p=5.683e-06 Score (logrank) test = 29.41 on 3 df, p=1.84e-06
```

- 10. What is the relation between the rate of ESRD between persons in remission and persons not?
- 11. What is the assumption about the two rates of remission? Refer to the figure with the three boxes you just made. (??).

3.1.2 Splitting the follow-up time

In order to explore the effect of remission on the rate of ESRD, we will split the data further into small pieces of follow-up. To this end we use the function splitLexis. The rates can then be modeled using a Poisson-model, and the shape of the underlying rates be explored. Furthermore, we can allow effects of both time since NRA and current age. To this end we will use splines, so we need the splines package, too.

12. First, split the follow-up time every month after entry, and make sure that the number of events and risk time is the same as before (use summary):

```
sLc <- splitLexis( Lc, "tfi", breaks=seq(0,30,1/12) )</pre>
 summary( Lc )
Transitions:
    To
From NRA Rem ESRD Records: Events: Risk time:
  NRA 24 29
               69
                        122
                                  98
                                         824.77
                                                      122
      0 24
               8
                        32
                                         259.90
  Rem
                                                      32
  Sum 24 53
               77
                        154
                                 106
                                        1084.67
                                                      125
 summary(sLc )
Transitions:
    Τо
From NRA
           Rem ESRD
                     Records:
                               Events: Risk time:
  NRA 9854
           29
                 69
                        9952
                               98
                                           824.77
                                                        122
  Rem
      0 3139
                 8
                         3147
                                    8
                                           259.90
                                                         32
  Sum 9854 3168
                 77
                        13099
                                   106
                                          1084.67
                                                        125
```

13. Now try to fit the Poisson-model corresponding to the Cox-model we fitted previously. The function ns() produces a model matrix corresponding to a piecewise cubic function, modeling the baseline hazard explicitly (think of the ns terms as the baseline hazard that is not visible in the Cox-model).

The outcome is 1 or 0 according to whether an event occurred or not, but sine a Poisson variate by definition is numerical, R will automatically coerce (change) a logical value to numeric; FALSE as 0 and TRUE as 1, so we can conveniently write:

```
library( splines )
 mp <- glm( lex.Xst=="ESRD" ~ ns( tfi, df=4 ) +</pre>
                  sex + I((doe-dob-40)/10) + (lex.Cst=="Rem"),
            offset = log(lex.dur),
            family = poisson,
             data = sLc)
 summary( mp )
Call:
glm(formula = lex.Xst == "ESRD" ~ ns(tfi, df = 4) + sex + I((doe -
    dob - 40)/10 + (lex.Cst == "Rem"), family = poisson, data = sLc,
    offset = log(lex.dur))
Deviance Residuals:
            1Q
                               3Q
   Min
                 Median
                                       Max
-0.2379 -0.1250 -0.0935 -0.0669
                                    3.7987
Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
                      -3.93862 0.72879 -5.404 6.51e-08
(Intercept)
ns(tfi, df = 4)1
                       2.10754
                                 0.72379
                                            2.912 0.003593
ns(tfi, df = 4)2
                       1.42695
                                 0.69738
                                            2.046 0.040741
ns(tfi, df = 4)3
                       3.49151
                                 1.66427
                                            2.098 0.035912
ns(tfi, df = 4)4
                       2.47260
                                 1.08261
                                          2.284 0.022376
                      -0.08043
                                0.27427 -0.293 0.769331
sexF
                                0.13714
                                          3.878 0.000105
I((doe - dob - 40)/10) 0.53187
lex.Cst == "Rem"TRUE -1.27858 0.38530 -3.318 0.000905
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 898.74 on 13098 degrees of freedom
Residual deviance: 853.54 on 13091 degrees of freedom
AIC: 1023.5
Number of Fisher Scoring iterations: 8
```

The ns function places knots at the quantiles of the variable, which may not be the most logica as the information is contained in the events, so the natural placement of knots would be at the quantiles of the event times. The Ns function in the Epi package automatically takes the smallest and the largest of the knots as boundary knots — the nuber of parameters is one less then the number of knots, so we use 5 knots:

```
t.kn \leftarrow with(\ subset(\ sLc,\ lex.Xst=="ESRD"), \\ quantile(\ tfi+lex.dur,\ 0:4/5\ ))
mp \leftarrow glm(\ lex.Xst=="ESRD" \ \ \ Ns(\ tfi,\ knots=t.kn\ ) + \\ sex + I((doe-dob-40)/10) + (lex.Cst=="Rem"), \\ offset = log(lex.dur), \\ family = poisson, \\ data = sLc\ )
summary(\ mp\ )
Call: \\ glm(formula = lex.Xst == "ESRD" \ \ \ Ns(tfi,\ knots = t.kn) + sex + \\ I((doe - dob - 40)/10) + (lex.Cst == "Rem"),\ family = poisson, \\ data = sLc,\ offset = log(lex.dur))
```

Deviance Residuals:

```
1Q
                  Median
                              30
                                      Max
-0.2573
        -0.1250
                 -0.0923
                         -0.0662
                                   3.7779
Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)
                     -3.59600 0.51242 -7.018 2.25e-12
Ns(tfi, knots = t.kn)1 1.95218 0.56113 3.479 0.000503
Ns(tfi, knots = t.kn)2 1.10038 0.41479 2.653 0.007982
Ns(tfi, knots = t.kn)3 2.30320 1.27583 1.805 0.071035
Ns(tfi, knots = t.kn)4 1.31387 0.32577 4.033 5.50e-05
                     -0.06981
                                0.27476 -0.254 0.799427
I((doe - dob - 40)/10) 0.53114
                                 0.13723 3.871 0.000109
lex.Cst == "Rem"TRUE -1.27896
                                0.38555 -3.317 0.000909
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 898.74 on 13098 degrees of freedom
Residual deviance: 852.46 on 13091 degrees of freedom
AIC: 1022.5
Number of Fisher Scoring iterations: 8
```

14. You can extract the parameters from the models using ci.lin or ci.exp try:

```
ci.lin( mp )
                         Estimate
                                     StdErr
(Intercept)
                      -3.59600346 0.5124153 -7.0177518 2.254666e-12 -4.6003190
Ns(tfi, knots = t.kn)1 1.95218041 0.5611331 3.4789971 5.032941e-04 0.8523797
Ns(tfi, knots = t.kn)2 1.10038347 0.4147933 2.6528474 7.981594e-03 0.2874034
Ns(tfi, knots = t.kn)3 2.30319625 1.2758310 1.8052518 7.103529e-02 -0.1973866
Ns(tfi, knots = t.kn)4 1.31387392 0.3257688 4.0331480 5.503459e-05 0.6753787
                      -0.06981245 0.2747561 -0.2540888 7.994269e-01 -0.6083245
I((doe - dob - 40)/10) 0.53114373 0.1372277 3.8705290 1.085995e-04 0.2621824
lex.Cst == "Rem"TRUE -1.27896398 0.3855503 -3.3172430 9.091051e-04 -2.0346287
                           97.5%
(Intercept)
                      -2.5916879
Ns(tfi, knots = t.kn)1 3.0519811
Ns(tfi, knots = t.kn)2
                       1.9133635
Ns(tfi, knots = t.kn)3 4.8037791
Ns(tfi, knots = t.kn)4 1.9523691
sexF
                       0.4686996
I((doe - dob - 40)/10) 0.8001050
lex.Cst == "Rem"TRUE -0.5232993
 ci.exp( mp )
                        exp(Est.)
                                        2.5%
                                                    97.5%
(Intercept)
                       0.02743314 0.01004863
                                               0.07489352
Ns(tfi, knots = t.kn)1 - 7.04402972 - 2.34522125 - 21.15721685
Ns(tfi, knots = t.kn)2 3.00531826 1.33296189
                                              6.77584104
Ns(tfi, knots = t.kn)3 10.00611340 0.82087321 121.97048700
Ns(tfi, knots = t.kn)4 3.72055898 1.96477698 7.04535900
sexF
                       0.93256870 0.54426203
                                               1.59791487
I((doe - dob - 40)/10) 1.70087654 1.29976361
                                               2.22577473
lex.Cst == "Rem"TRUE
                       0.27832550 0.13072902
                                               0.59256227
```

ci.exp(mp, subset=c("sex","dob","Cst"), pval=TRUE)

Compare with the estimates from the Cox-model. Use:

```
ci.exp( m1 )
                       exp(Est.)
                                       2.5%
                                               97.5%
                       0.9461646 0.5519334 1.621985
sexF
I((doe - dob - 50)/10) 1.6852196 1.2895097 2.202360
lex.Cst == "Rem"TRUE
                       0.2829710 0.1330996 0.601599
 ci.exp( mp, subset=c("sex","dob","Cst") )
                       exp(Est.)
                                      2.5%
                                               97.5%
sexF
                       0.9325687 0.544262 1.5979149
I((doe - dob - 40)/10) 1.7008765 1.299764 2.2257747
lex.Cst == "Rem"TRUE
                       0.2783255 0.130729 0.5925623
 ci.exp( mp, subset=c("sex","dob","Cst") ) / ci.exp( m1 )
                       exp(Est.)
                                       2.5%
                                                97.5%
sexF
                       0.9856305 0.9861009 0.9851603
I((doe - dob - 40)/10) 1.0092907 1.0079518 1.0106315
lex.Cst == "Rem"TRUE
                       0.9835830 0.9821891 0.9849789
```

What do you conclude about the models?

15. You can visualize the spline term using termplot, try:

```
termplot( mp, terms=1 )
```

... which is not a terribly informative plot

16. termplot does not give you the absolute level of the underlying rates because it bypasses the intercept. If you explicitly include the intercept in the baseline split you can use Termplot from the Epi package to get estimates on the rate scale for a reference person (in units of events per 100 years):

How would you describe this rate function in plain words? And what is the scale of the y-axis.

Annotate the axes of the plot accordingly — consult the help page of Termplot.

17. Apart from the baseline timescale, time since NRA, time since remission might be of interest in describing the mortality rate. However this is only relevant for persons who actually have a remission, so start by checking how many events there are in this group:

```
summary( sLc )
Transitions:
    Tο
From NRA
         Rem ESRD Records: Events: Risk time: Persons:
 NRA 9854
         29 69
                  9952
                           98 824.77
                                            122
     0 3139
 Rem
               8
                      3147
                               8
                                     259.90
                                                 32
 Sum 9854 3168
               77
                     13099
                              106
                                    1084.67
                                                 125
```

How many go in remission, and how many deaths are in this group?

18. With this rather limited number of events we can certainly not expect to be able to model anything more complicated than a linear trend with time since remission. Two parameters on 8 events is actually pretty far-fetched.

The variable we want to have in the model is current date (per) minus date of remission (dor): per-dor), but only positive values of it. This can be fixed by using pmax(), but we must also deal with all those who have missing values, so we use the construct:

```
pmax( per-dor, 0, na.rm=TRUE )
```

Make sure that you understand what goes on here.

19. We can now expand the model with this variable:

```
sLc <- transform( \ sLc, \ tfr = pmax( \ (per-dor)/10, \ 0, \ na.rm=TRUE \ ) \ ) \\ mPx <- glm( \ lex.Xst="ESRD" \ ^-1 + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( \ tfi, \ knots=t.kn, \ intercept=TRUE \ ) + Ns( 
                                                          sex + I((age-tfi-40)/10) + (lex.Cst=="Rem") + tfr,
                                                  offset = log(lex.dur/100),
                                                  family = poisson,
                                                         data = sLc )
    round( ci.exp( mPx ), 3 )
                                                                                                                                                            exp(Est.)
                                                                                                                                                                                                     2.5%
                                                                                                                                                                                                                                    97.5%
Ns(tfi, knots = t.kn, intercept = TRUE)1
                                                                                                                                                                                                      1.466
                                                                                                                                                                        4.789
                                                                                                                                                                                                                                15.641
Ns(tfi, knots = t.kn, intercept = TRUE)2
                                                                                                                                                                        17.935
                                                                                                                                                                                                      7.985
                                                                                                                                                                                                                                40.283
Ns(tfi, knots = t.kn, intercept = TRUE)3
                                                                                                                                                                        5.581
                                                                                                                                                                                                   2.649 11.760
                                                                                                                                                                        51.347 13.438 196.202
Ns(tfi, knots = t.kn, intercept = TRUE)4
Ns(tfi, knots = t.kn, intercept = TRUE)5
                                                                                                                                                                          6.427
                                                                                                                                                                                                      3.368
                                                                                                                                                                                                                               12.266
sexM
                                                                                                                                                                           1.079
                                                                                                                                                                                                     0.628
                                                                                                                                                                                                                                   1.853
sexF
                                                                                                                                                                           1.000 1.000
                                                                                                                                                                                                                                    1.000
I((age - tfi - 40)/10)
                                                                                                                                                                           1.703 1.302
                                                                                                                                                                                                                                     2.229
lex.Cst == "Rem"TRUE
                                                                                                                                                                            0.310 0.097
                                                                                                                                                                                                                                     0.989
                                                                                                                                                                            0.847 0.210
                                                                                                                                                                                                                                     3.412
tfr
    Termplot( mPx, terms=1 )
```

- 20. Is the effect significant? Can a substantial effect of time since remission be ruled out?
- 21. What is the test of this parameter traditionally called? What is the null and what is the alternative of this test?

3.1.3 Prediction in a multistate model

This part of the practical is about making proper statements about the survival and the disease probabilities. But in order to do this we must know not only how the occurrence of remission influences the rate of death/ESRD, but we must also model the occurrence rate of remission itself.

The following exercise will be quite similar to the example in the help file for simLexis (which you should read now!).

22. The rates of ESRD were modelled by a Poisson model with effects of age and time since NRA — in the model mp. But in the modelling of the remission rates transition from "NRA" to "Rem", the number of events is rather small, so we restrict the variables in this model to only time since NRA and sex. Also remember, only the records that relate to the "NRA" state can be used:

```
mr <- glm( lex.Xst=="Rem" ~ ns( tfi, knots=t.kn ) + sex,</pre>
            offset = log(lex.dur),
            family = poisson,
              data = subset( sLc, lex.Cst=="NRA" ) )
 ci.exp( mr, pval=TRUE )
                         exp(Est.)
                                            2.5%
                                                        97.5%
(Intercept)
                        0.05606873 0.0155421035
                                                    0.2022701 1.075954e-05
ns(tfi, knots = t.kn)1 1.56250187 0.1758966092
                                                   13.8798132 6.888024e-01
ns(tfi, knots = t.kn)2 0.12621768 0.0105935727
                                                    1.5038272 1.015853e-01
ns(tfi, knots = t.kn)3 \ 0.61154986 \ 0.0701435838
                                                    5.3318238 6.562519e-01
ns(tfi, knots = t.kn)4 0.97532990 0.0280655093
                                                   33.8945715 9.889910e-01
ns(tfi, knots = t.kn)5 \ 0.08049791 \ 0.0004655089
                                                   13.9200643 3.378924e-01
ns(tfi, knots = t.kn)6 0.65167781 0.0002104090 2018.3737166 9.168447e-01
sexF
                        2.64124116 1.2658909206
                                                    5.5108657 9.645522e-03
```

23. If we want to predict the probability of being in each of the three states using these estimated rates, we can either do analytical calculations of the probabilities from the estimated rates, or we can *simulate* the life course through a model using the estimated rates. That will give a simulated cohort (in the form of a Lexis object), and we can then just count the number of persons in each state at each of a set of time points.

This is accomplished using the function simLexis. The input to this is the initial status of the persons whose life-course we shall simulate, and the transition rates in suitable form:

• Suppose we want predictions for men aged 50 at NRA. The input is in the form of a Lexis object (where lex.dur and lex.Xst will be ignored). Note that in order to carry over the time.scales and the time.since attributes, we construct the input object using subset to select columns, and NULL to select rows (see the example in the help file for simLexis):

```
inL <- subset( sLc, select=1:11 )[NULL,]
str( inL )
Classes 'Lexis' and 'data.frame': 0 obs. of 11 variables:
$ lex.id : int
$ per : num</pre>
```

```
$ age
           : num
$ tfi
           : num
$ lex.dur: num
$ lex.Cst: Factor w/ 3 levels "NRA", "Rem", "ESRD":
$ lex.Xst: Factor w/ 3 levels "NRA", "Rem", "ESRD":
$ id
           : num
           : Factor w/ 2 levels "M", "F":
$ sex
$ dob
           : num
$ doe
           : num
- attr(*, "breaks")=List of 3
  .. $ per: NULL
  ..$ age: NULL
  ..$ tfi: num 0 0.0833 0.1667 0.25 0.3333 ...
- attr(*, "time.scales")= chr "per" "age" "tfi"
                                   - attr(*, "time.since")= chr
timeScales(inL)
[1] "per" "age" "tfi"
inL[1,"lex.id"] <- 1</pre>
inL[1,"per"] <- 2000
inL[1, "age"] <- 50
inL[1,"tfi"] <- 0
inL[1,"lex.Cst"] <- "NRA"
inL[1,"lex.Cst"] <- NA
inL[1,"lex.Xst"] <- NA
inL[1,"lex.dur"] <- NA
inL[1,"sex"] <- "M"
inL[1,"doe"] <- 2000
inL[1,"dob"] <- 1950
inL
  lex.id per age tfi lex.dur lex.Cst lex.Xst id sex dob doe
1
        1 2000
                50
                      0
                               NA
                                       NR.A
                                               <NA> NA
                                                           M 1950 2000
```

• The other input for the simulation is the transitions, which is a list with an element for each transient state (that is "NRA" and "Rem"), each of which is again a list with names equal to the states that can be reached from the transient state. The content of the list will be glm objects, in this case the models we just fitted, describing the transition rates:

With this as input we can now generate a cohort, using N=10 to simulate life course of 10 persons (with identical starting values):

```
( iL <- simLexis( Tr, inL, N=10 ) )
   lex.id
                                  tfi
                                         lex.dur lex.Cst lex.Xst id sex
                                                                         dob
                                                                               doe cens
               per
                         age
        1 2000.000 50.00000 0.000000
                                       3.9537448
                                                      NRA
                                                             ESRD NA
                                                                       M 1950 2000 2020
        2 2000.000 50.00000 0.000000
                                                             ESRD NA
                                                                       M 1950 2000 2020
                                       3.5109129
                                                      NRA
3
        3 2000.000 50.00000 0.000000
                                       7.7976187
                                                      NRA
                                                              Rem NA
                                                                       M 1950 2000 2020
        3 2007.798 57.79762 7.797619 12.2023813
                                                              Rem NA
                                                                       M 1950 2000 2020
                                                      Rem
5
        4 2000.000 50.00000 0.000000
                                                                       M 1950 2000 2020
                                       5.8765578
                                                      NRA
                                                             ESRD NA
                                                             ESRD NA
6
        5 2000.000 50.00000 0.000000
                                       3.5747715
                                                      NRA
                                                                       M 1950 2000 2020
7
        6 2000.000 50.00000 0.000000
                                                             ESRD NA
                                                                       M 1950 2000 2020
                                       0.4049354
                                                      NRA
        7 2000.000 50.00000 0.000000
8
                                       5.8315212
                                                      NRA
                                                             ESRD NA
                                                                       M 1950 2000 2020
9
        8 2000.000 50.00000 0.000000
                                       6.1983270
                                                      NRA
                                                             ESRD NA
                                                                       M 1950 2000 2020
10
        9 2000.000 50.00000 0.000000
                                       2.3599612
                                                      NRA
                                                             ESRD NA
                                                                       M 1950 2000 2020
       10 2000.000 50.00000 0.000000 3.9291606
                                                      NRA
                                                             ESRD NA
                                                                       M 1950 2000 2020
```

```
summary( iL )
Transitions:
    To
From NRA Rem ESRD Records: Events: Risk time: Persons:
 NRA 0 1 9 10
                         10
                                    43.44
          1
              0
                      1
                              0
                                    12.20
 Rem
                                                1
 Sum
                      11
                             10
                                    55.64
                                               10
```

24. Now generate the life course of 10,000 persons, and look at the summary. The system.time command is jus to tell you how long it took, you may want to start with 1000 just to see how long that takes.

```
system.time(
 sM <- simLexis( Tr, inL, N=10000 ) )</pre>
  user system elapsed
 16.266
        0.192 16.466
 summary( sM )
Transitions:
    To
From NRA Rem ESRD Records: Events: Risk time:
  NRA 28 1404 8568
                       10000
                              9972
                                        56134.77
                                                     10000
  Rem 0 391 1013
                        1404
                                 1013
                                        14345.93
                                                     1404
  Sum 28 1795 9581
                       11404
                                10985
                                        70480.70
                                                     10000
```

Why are there so many ESRD-events in the resulting data set?

25. Now we want to count how many persons are present in each state at each time for the first 10 years after entry (which is at age 50). This can be done by using nState:

```
nSt <- nState( sM, at=seq(0,10,0.1), from=50, time.scale="age" )</pre>
head( nSt )
      State
               Rem ESRD
when
         NRA
       10000
                0
                       0
  50.1 9915
                43
                       42
  50.2
        9815
               104
                       81
                      127
  50.3
        9719
               154
  50.4
       9632
               200
                      168
  50.5 9557
               225
                      218
```

26. Once we have the counts of persons in each state at the designated time points, we compute the cumulative fraction over the states, arranged in order given by perm:

```
pp <- pState( nSt, perm=1:3 )
head( pp )</pre>
```

```
State
when
          NRA
                  Rem ESRD
       1.0000 1.0000
  50.1 0.9915 0.9958
                         1
  50.2 0.9815 0.9919
                         1
  50.3 0.9719 0.9873
                         1
  50.4 0.9632 0.9832
                         1
  50.5 0.9557 0.9782
                         1
 tail(pp)
      State
when
          NRA
                  Rem ESRD
  59.5 0.1470 0.2371
  59.6 0.1430 0.2329
                         1
  59.7 0.1392 0.2287
                         1
  59.8 0.1362 0.2254
                         1
  59.9 0.1330 0.2220
                         1
  60
       0.1295 0.2186
                         1
```

27. Try to plot the cumulative probabilities using the plot method for pState objects:

```
plot( pp )
```

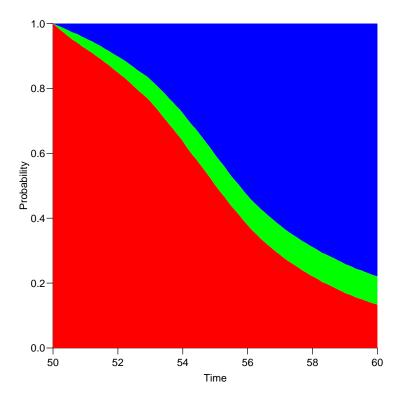


Figure 3.3: Standard plot of state occupancy probabilities.

28. A quantity of particular interest would be how many patients actually get a remission. This is not deductible from the plot just shown, because those who get ESRD are not subdivided according to whether they have a remission prior to ESRD.

The simplest way to doctor that is to modify the simulated object (sM in the above notation), so that those exiting to "ESRD" from "Rem" are counted in a separate state. We must also change the formal set of levels of lex.Cst:

```
xM <- transform( sM, lex.Xst = factor( ifelse( lex.Xst=="ESRD" & lex.Cst=="Rem",
                                                "ESRD(Rem)",
                                                as.character(lex.Xst)),
                                        levels=c("NRA","Rem","ESRD(Rem)","ESRD") ),
                      lex.Cst = factor( as.character(lex.Cst),
                                        levels=c("NRA", "Rem", "ESRD(Rem)", "ESRD") ) )
 summary( sM )
Transitions:
    To
From NRA Rem ESRD
                    Records: Events: Risk time:
  NRA 28 1404 8568
                       10000
                                 9972
                                         56134.77
                                 1013
      0 391 1013
                        1404
                                        14345.93
                                                      1404
  Sum 28 1795 9581
                       11404
                                 10985
                                        70480.70
                                                     10000
 summary( xM )
Transitions:
    To
From NRA Rem ESRD(Rem) ESRD Records:
                                        Events: Risk time:
                                                            Persons:
  NRA 28 1404
                      0 8568
                                 10000
                                            9972
                                                   56134.77
                                                                10000
  Rem 0 391
                    1013 0
                                  1404
                                            1013
                                                   14345.93
                                                                1404
  Sum 28 1795
                   1013 8568
                                  11404
                                           10985
                                                   70480.70
                                                               10000
 boxes(xM, boxpos=TRUE, show.BE=TRUE, scale.R=100)
```

29. Having done this, try to compute the number of persons in each of the 4 states, and the cumulative proportions to be plotted:

```
xSt <- nState( xM, at=seq(0,10,0.1), from=50, time.scale="age" )
xp <- pState( xSt, perm=1:4 )</pre>
head(xp)
     State
when
         NRA
                 Rem ESRD(Rem) ESRD
      1.0000 1.0000
                     1.0000
                                  1
  50.1 0.9915 0.9958
                        0.9958
                                  1
  50.2 0.9815 0.9919
                        0.9919
                                  1
  50.3 0.9719 0.9873
                        0.9873
                                  1
  50.4 0.9632 0.9832
                        0.9832
                                  1
  50.5 0.9557 0.9782
                        0.9782
plot( xp, col=rev(c("pink","limegreen","forestgreen","red")), xlab="Age" )
 lines( as.numeric(rownames(xp)), xp[,"Rem"], lwd=4 )
```

What is the probability that a 50-year old man with NRA sees a remission from NRA during the next 10 yezrs?

- 30. Make the same calculations for a 60-year old woman.
- 31. Normally you would know that a split of the absorbing "ESRD" state according to the preceding state and so define this in the cutLexis function, using split.states. At the same time it is also possible to define a new timescale using new.scale, defined as time since entry to the new state:

```
Lc <- cutLexis( Lr, cut = Lr$dor, # where to cut follow up
              timescale = "per", # the timescale that "dor" refers to
new.state = "Rem", # name of the new state
       precursor.states = "NRA", # which states are less severe
              new.scale = "tfr", # define a new timescale as time since Rem
            split.states = TRUE ) # subdivide non-precursor states
str( Lc )
Classes 'Lexis' and 'data.frame':
                                       154 obs. of 15 variables:
        : num 1996 1990 1990 1988 1995 ...
         : num 28.1 30.2 30.5 25.8 44.5 ...
        : num 0 0 0.279 0 0 ...
$ tfi
        : num NA NA O NA NA O NA O NA NA ...
$ lex.dur: num 1.081 0.279 6.322 5.393 0.473 ...
$ lex.Cst: Factor w/ 4 levels "NRA", "Rem", "ESRD", ...: 1 1 2 1 1 2 1 2 1 1 ...
$ lex.Xst: Factor w/ 4 levels "NRA", "Rem", "ESRD", ...: 3 2 4 3 2 2 2 2 3 3 ...
$ lex.id : int 1 2 2 3 4 4 5 5 6 7 ...
$ id : num 17 26 26 27 33 33 42 42 46 47 ...
         : Factor w/ 2 levels "M", "F": 1 2 2 2 1 1 2 2 2 1 ...
        : num 1968 1959 1959 1962 1951 ...
$ dob
        : num 1996 1990 1990 1988 1995 ...
$ doe
$ dor
        : num NA 1990 1990 NA 1996 ...
        : num 1997 1996 1996 1993 2004 ...
$ dox
$ event : num 2 1 1 3 0 0 0 0 2 1 ...
 - attr(*, "breaks")=List of 4
 .. $ per: NULL
 ..$ age: NULL
 ..$ tfi: NULL
 ..$ tfr: NULL
 - attr(*, "time.scales")= chr "per" "age" "tfi" "tfr"
- attr(*, "time.since")= chr "" "Rem"
 # source("/home/bendix/stat/R/lib.src/Epi/pkg/R/summary.Lexis.r")
 # summary( Lc, S=T, scale=100 )
summary( Lc )
Transitions:
    To
From NRA Rem ESRD ESRD(Rem) Records: Events: Risk time: Persons:
 NRA 24 29 69 0
                                  122
                                           98 824.77
                                                                122
 Rem 0 24
               0
                          8
                                  32
                                            8
                                                   259.90
                                                                32
 Sum 24 53
               69
                         8
                                  154
                                           106
                                                                125
                                                  1084.67
 boxes(Lc, boxpos=list(x=c(20,80,20,80), y=c(80,80,20,20)),
            scale.R=100, show.BE=TRUE )
 sLc <- splitLexis( Lc, "tfi", breaks=seq(0,30,1/12) )</pre>
 summary( Lc )
Transitions:
From NRA Rem ESRD ESRD(Rem) Records: Events: Risk time:
                   0
 NRA 24 29
               69
                                  122
                                           98
                                                  824.77
                                                                122
      0 24
                          8
                                  32
                                            8
                                                   259.90
                                                                 32
 Rem
                0
                    8
 Sum 24 53
               69
                                 154
                                       106
                                                 1084.67
                                                                125
summary( sLc )
Transitions:
From NRA Rem ESRD ESRD(Rem) Records: Events: Risk time: Persons:
```

```
NRA 9854
             29
                  69
                              0
                                     9952
                                                98
                                                       824.77
                                                                     122
         0 3139
                  0
                             8
                                     3147
                                                8
                                                        259.90
                                                                      32
  Sum 9854 3168
                  69
                             8
                                    13099
                                               106
                                                      1084.67
                                                                     125
 head( subset( sLc, lex.id==2 )[,1:8], 8 )
                                                      lex.dur lex.Cst lex.Xst
                                    tfi
                                               tfr
   lex.id
               per
                        age
14
        2 1989.535 30.22895 0.00000000
                                                NA 0.08333333
                                                                   NRA
        2 1989.618 30.31229 0.08333333
15
                                                NA 0.08333333
                                                                   NRA
                                                                           NRA
        2 1989.702 30.39562 0.16666667
                                                NA 0.08333333
                                                                   NRA
                                                                           NRA
        2 1989.785 30.47895 0.25000000
                                                                   NRA
                                                                           Rem
                                                NA 0.02891855
        2 1989.814 30.50787 0.27891855 0.00000000 0.05441478
                                                                   Rem
                                                                           Rem
        2 1989.868 30.56229 0.33333333 0.05441478 0.08333333
                                                                   Rem
                                                                           Rem
        2 1989.952 30.64562 0.41666667 0.13774812 0.08333333
                                                                           R.em
        2 1990.035 30.72895 0.50000000 0.22108145 0.08333333
                                                                           Rem
 tail( subset( sLc, lex.id==2 )[,1:8], 3 )
                                                  lex.dur lex.Cst
                                                                     lex.Xst
   lex.id
                                  tfi
                                           tfr
               per
        2 1995.952 36.64562 6.416667 6.137748 0.08333333
                                                              R.em
                                                                         R.em
        2 1996.035 36.72895 6.500000 6.221081 0.08333333
93
                                                              Rem
                                                                         Rem
        2 1996.118 36.81229 6.583333 6.304415 0.01728268
                                                              Rem ESRD(Rem)
 ( f1 <- levels(Lc)[3:4] )
[1] "ESRD"
                "ESRD(Rem)"
 mp <- glm( lex.Xst %in% fl ~ ns( tfi, df=4 ) +</pre>
                               sex + I((age-tfi-40)/10) + (lex.Cst=="Rem"),
            offset = log(lex.dur/100),
            family = poisson,
              data = sLc)
 # the timescale tfr must be given some value for time before Rem
 sLc$tfr <- pmax( 0, sLc$tfr, na.rm=TRUE )</pre>
 head( subset( sLc, lex.id==2 )[,1:8], 8 )
   lex.id
                                    tfi
                                               tfr
                                                      lex.dur lex.Cst lex.Xst
                        age
14
        2 1989.535 30.22895 0.00000000 0.00000000 0.08333333
        2 1989.618 30.31229 0.08333333 0.00000000 0.08333333
                                                                           NRA
15
        2 1989.702 30.39562 0.16666667 0.00000000 0.08333333
16
                                                                   NRA
                                                                           NRA
        2 1989.785 30.47895 0.25000000 0.00000000 0.02891855
                                                                   NRA
                                                                           Rem
17
        2 1989.814 30.50787 0.27891855 0.00000000 0.05441478
                                                                   Rem
                                                                           Rem
18
        2 1989.868 30.56229 0.33333333 0.05441478 0.08333333
19
                                                                   Rem
                                                                           Rem
        2 1989.952 30.64562 0.41666667 0.13774812 0.08333333
20
                                                                   Rem
                                                                           Rem
21
        2 1990.035 30.72895 0.50000000 0.22108145 0.08333333
                                                                   Rem
                                                                           Rem
 mr \leftarrow glm(lex.Xst=="Rem" ~ ns(tfi, df=4) + sex,
            offset = log(lex.dur),
            family = poisson,
              data = subset( sLc, lex.Cst=="NRA" ) )
 ci.exp( mr, pval=TRUE )
                  exp(Est.)
                                    2.5%
                                               97.5%
(Intercept)
                 0.03606128 \ 0.011013035
                                           0.1180797 4.016649e-08
ns(tfi, df = 4)1 0.43778959 0.094970457
                                           2.0180984 2.894125e-01
ns(tfi, df = 4)2 1.15591640 0.112100187 11.9191838 9.031269e-01
ns(tfi, df = 4)3 0.57520635 0.017327786 19.0943229 7.569600e-01
ns(tfi, df = 4)4 0.69162506 0.003446815 138.7788899 8.915761e-01
                 2.63407462 1.261956986
                                         5.4980868 9.889849e-03
```

```
inL <- subset( sLc, select=1:10 )[NULL,]</pre>
 str(inL)
Classes 'Lexis' and 'data.frame': 0 obs. of 10 variables:
 $ lex.id : int
 $ per : num
 $ age
         : num
 $ tfi
         : num
 $ tfr
         : num
 $ lex.dur: num
 $ lex.Cst: Factor w/ 4 levels "NRA", "Rem", "ESRD",...:
 $ lex.Xst: Factor w/ 4 levels "NRA", "Rem", "ESRD",...:
       : num
 $ sex
         : Factor w/ 2 levels "M", "F":
 - attr(*, "breaks")=List of 4
  ..$ per: NULL
  ..$ age: NULL
  ..$ tfi: num 0 0.0833 0.1667 0.25 0.3333 ...
  ..$ tfr: NULL
 - attr(*, "time.scales")= chr "per" "age" "tfi" "tfr" - attr(*, "time.since")= chr "" "" "Rem"
 timeScales(inL)
[1] "per" "age" "tfi" "tfr"
 inL[1,"lex.id"] <- 1
inL[1, 'lex.'ld'] <- 1
inL[1, "per"] <- 2000
inL[1, "age"] <- 50
inL[1, "tfi"] <- 0
inL[1, "lex.'Cst"] <- "NRA"
 inL[1,"lex.Xst"] <- NA
inL[1,"lex.dur"] <- NA</pre>
 inL[1,"sex"] <- "M"</pre>
  lex.id per age tfi tfr lex.dur lex.Cst lex.Xst id sex
       1 2000 50
                              NA
                                     NRA <NA> NA
                   O NA
 Tr <- list( "NRA" = list( "Rem" = mr,</pre>
                            "ESRD" = mp ),
             "Rem" = list( "ESRD(Rem)" = mp ) )
 ( iL <- simLexis( Tr, inL, N=10 ) )
                                  tfi tfr lex.dur lex.Cst
                                                                lex.Xst id sex cens
   lex.id
               per
                       age
                                                                   ESRD NA M 2020
        1 2000.000 50.0000 0.0000000 NA 4.4229390 NRA
1
        2 2000.000 50.0000 0.0000000 NA 0.2273966
2
                                                         NRA
                                                                    Rem NA M 2020
3
        Rem ESRD(Rem) NA M 2020
4
        3 2000.000 50.0000 0.0000000 NA 8.2789058
                                                         NRA
                                                                   ESRD NA M 2020
5
        4 2000.000 50.0000 0.0000000 NA 8.9320274
                                                         NRA
                                                                   ESRD NA M 2020
       5 2000.000 50.0000 0.0000000 NA 17.8870698
                                                         NRA
                                                                   ESRD NA M 2020
7
       6 2000.000 50.0000 0.0000000 NA 4.2673109
                                                         NRA
                                                                   ESRD NA M 2020
8
       7 2000.000 50.0000 0.0000000 NA 14.5124058
                                                         NRA
                                                                   ESRD NA M 2020
9
       8 2000.000 50.0000 0.0000000 NA 5.4584126
                                                                   ESRD NA M 2020
                                                         NRA
       9 2000.000 50.0000 0.0000000 NA 7.8017705
                                                                   ESRD NA
                                                                            M 2020
10
                                                         NRA
       10 2000.000 50.0000 0.0000000 NA 17.0273563
                                                                   ESRD NA M 2020
                                                         NRA
 summary( iL )
```

```
Transitions:
    To
From NRA Rem ESRD ESRD(Rem) Records: Events: Risk time: Persons:
                                     10 88.82
         1 9 0 10
                        1
 Rem
       0
           0
               0
                                 1
                                          1
                                                  6.92
                                                              1
 Sum
       0
           1
               9
                         1
                                 11
                                         11
                                                 95.73
                                                             10
system.time(
 sM <- simLexis( Tr, inL, N=10000, t.range=25, n.int=251 ) )</pre>
  user system elapsed
        0.456 25.130
24.655
summary( sM )
Transitions:
    Тο
From NRA Rem ESRD ESRD(Rem) Records: Events: Risk time: Persons:
 NRA 1 1438 8561 0 10000
                                      9999
                                               56556.84
                                                            10000
 Rem 0 148
              0
                       1290
                                1438
                                         1290
                                               15638.24
                                                            1438
 Sum 1 1586 8561
                      1290
                               11438
                                        11289
                                               72195.08
                                                           10000
nSt <- nState( sM, at=seq(0,24,0.1), from=50, time.scale="age" )</pre>
head( nSt )
     State
             Rem ESRD ESRD(Rem)
when
       NRA
 50
      10000
             0 0 0
 50.1 9928
              41
                    31
 50.2 9858
              81
                    61
                              0
 50.3 9787
              116
                   97
                              0
 50.4 9727
            144 129
                              0
 50.5 9659 182 159
pp \leftarrow pState(nSt, perm=c(1,2,4,3))
head(pp)
     State
               Rem ESRD(Rem) ESRD
when
        NRA
      1.0000 1.0000 1.0000
 50.1 0.9928 0.9969
                      0.9969
                               1
 50.2 0.9858 0.9939
                     0.9939
                               1
 50.3 0.9787 0.9903
                    0.9903
                               1
                              1
 50.4 0.9727 0.9871 0.9871
 50.5 0.9659 0.9841 0.9841
                              1
 tail( pp )
     State
      NRA
              Rem ESRD(Rem) ESRD
when
 73.5 2e-04 0.0207
                   0.144 1
 73.6 2e-04 0.0204
                      0.144
                               1
 73.7 2e-04 0.0202
                      0.144
                              1
 73.8 2e-04 0.0199
                      0.144
 73.9 2e-04 0.0196
                      0.144
                               1
 74 2e-04 0.0191
                      0.144
                              1
plot( pp )
 # Two colors and the corresponding pale ones for the dead states
 clr <- c("limegreen", "orange")</pre>
 col2rgb(clr)
```

```
[,1] [,2]
red 50 255
green 205 165
blue 50 0

cl4 <- cbind(col2rgb(clr),col2rgb(clr)/2+255/2)[,c(1,2,4,3)]
cl4 <- rgb( t(cl4), max=255 )
# Nicer plot
plot( pp, col=cl4, xlab="Age" )
lines( as.numeric(rownames(pp)), pp[,2], lwd=2 )
```

3.2 Time-splitting, time-scales and SMR: Diabetes in Denmark

This exercise is using data from the National Danish Diabetes register. There is a random sample of 10,000 records from this in the Epi package. Actually there are two data sets, we shall use the one with only cases of diabetes diagnosed after 1995, see the help page for DMlate.

This is of interest because it is only for these where the data of diagnosis is certain, and hence for whom we can compute the duration of diabetes during follow-up.

The exercise is about assessing how mortality depends age, calendar time and duration of diabetes. And how to understand and compute SMR, and assess how it depends on these factors as well.

1. First, we load the Epi package and the dataset, and take a look at it:

```
> options( width=90 )
> library( Epi )
> data( DMlate )
> str( DMlate )
'data.frame':
                    10000 obs. of 7 variables:
$ sex : Factor w/ 2 levels "M","F": 2 1 2 2 1 2 1 1 2 1 ...
$ dobth: num 1940 1939 1918 1965 1933 ...
$ dodm : num 1999 2003 2005 2009 2009 ...
$ dodth: num NA NA NA NA NA ...
$ dooad: num NA 2007 NA NA NA ...
$ doins: num NA ...
$ dox : num 2010 2010 2010 2010 2010 ...
> head( DMlate )
       sex
              dobth
                        dodm
                                dodth
                                         dooad doins
                                                          dox
50185
       F 1940.256 1998.917
                                  NA
                                            NA
                                                  NA 2009.997
307563
        M 1939.218 2003.309
                                   NA 2007.446
                                                  NA 2009.997
294104
        F 1918.301 2004.552
                                  NA
                                           NA
                                                  NA 2009.997
336439
        F 1965.225 2009.261
                                  NA
                                            NA
                                                  NA 2009.997
245651
        M 1932.877 2008.653
                                  NA
                                           NA
                                                  NA 2009.997
       F 1927.870 2007.886 2009.923
                                            NA
                                                  NA 2009.923
216824
> summary( DMlate )
```

```
dobth
                              dodm
                                            dodth
                                                            dooad
                                                                            doins
sex
M:5185
         Min.
               :1898
                        Min.
                              :1995
                                        Min.
                                              :1995
                                                        Min.
                                                               :1995
                                                                        Min.
                                                                               :1995
F:4815
         1st Qu.:1930
                         1st Qu.:2000
                                        1st Qu.:2002
                                                        1st Qu.:2001
                                                                        1st Qu.:2001
         Median:1941
                        Median:2004
                                        Median:2005
                                                        Median:2004
                                                                        Median:2005
                :1942
                                                :2005
                                                                               :2004
         Mean
                        Mean
                                :2003
                                        Mean
                                                        Mean
                                                                :2004
                                                                        Mean
         3rd Qu.:1951
                         3rd Qu.:2007
                                        3rd Qu.:2008
                                                        3rd Qu.:2007
                                                                        3rd Qu.:2007
                :2008
                                :2010
                                               :2010
                                                        Max.
                                                               :2010
                                                                               :2010
         Max.
                        Max.
                                        Max.
                                                                        Max.
                                        NA's
                                                        NA's
                                                               :4503
                                                                       NA's
                                                :7497
                                                                               :8209
     dox
Min.
      :1995
1st Qu.:2010
Median:2010
       :2009
Mean
3rd Qu.:2010
Max.
       :2010
```

2. We then set up the dataset as a Lexis object with age, calendar time and duration of diabetes as timescales, and date of death as event.

In the dataset we have a date of exit dox which is either the day of censoring or the date of death:

So we can set up the Lexis object by specifying the timescales and the exit status via !is.na(dodth):

The 4 persons are persons that have identical date of diabetes and date of death.

We can get an overview of the data by using the summary function on the object:

```
> summary( LL )
Transitions:
    To
From Alive Dead Records: Events: Risk time: Persons:
    Alive 7497 2499 9996 2499 54273.27 9996
> head( LL )
```

```
P dur
                                lex.dur lex.Cst lex.Xst lex.id sex
                                                                     dobth
50185 58.66119 1998.917
                         0 11.0800821
                                        Alive
                                                 Alive 1 F 1940.256 1998.917
                                         Alive Alive 2
Alive Alive 3
307563 64.09035 2003.309
                         0 6.6885695
                                                                M 1939.218 2003.309
294104 86.25051 2004.552
                                                               F 1918.301 2004.552
                         0 5.4455852
                                         Alive Alive 4 F 1965.225 2009.261
Alive Alive 5 M 1932.877 2008.653
Alive Dead 6 F 1927.870 2007.886
                         0 0.7364819
336439 44.03559 2009.261
245651 75.77550 2008.653
                         0 1.3442847
216824 80.01643 2007.886
                          0 2.0369610
         dodth dooad doins
                                   dox
          NA
50185
                   NA NA 2009.997
           NA 2007.446 NA 2009.997
307563
294104
           NA NA NA 2009.997
336439
            NA
                     NA NA 2009.997
245651
           NA
                    NA NA 2009.997
216824 2009.923
                    NA
                           NA 2009.923
```

3. A very crude picture of the mortality by sex can be obtained by the stat.table function:

So not surprising, we see that men have a higher mortality than women.

4. We now want to assess how mortality depends on age, calendar time and duration. In principle we could split the follow-up along all three time scales, but in practice it would be sufficient to split it along one of the time-scales and then just use the value of each of the time-scales at the left endpoint of the intervals.

We note that the total follow-up time was some 54,000 person-years, so if we split the follow-up in 6-month intervals we get a bit more than 110,000 records:

```
> SL <- splitLexis( LL, breaks=seq(0,125,1/2), time.scale="A" )
> summary( SL )
Transitions:
    To
        Alive Dead Records: Events: Risk time: Persons:
  Alive 115974 2499 118473
                                2499 54273.27
                                                     9996
> summary( LL )
Transitions:
       Alive Dead Records: Events: Risk time:
From
  Alive 7497 2499
                   9996
                               2499
                                      54273.27
                                                    9996
```

We see that the number of records have increased, but the number of persons, events and person-years is still the same as in LL

5. We now use this dataset to estimate models with age-specific mortality curves for men and women separately, using natural splines (the function ns from the splines package).

Here we are modeling the follow-up (events ((lex.Xst=="Dead")) and person-years (lex.dur)) as a non-liner function of age — represented by the spline function ns.

6. From these objects we could get the estimated log-rates by using predict, by supplying a data frame of values for the variables corresponding to the predictor variables in the model.

The default predict.glm function is a bit clunky as it gives the prediction and the standard errors of these in two different elements of a list, so in Epi there is a wrapper function ci.pred that uses this and computes predicted rates and confidence limits for these.

Note that lex.dur is a covariate too; by putting this to 1000 we get the rates in units of deaths per 1000 PY:

7. We can then plot the predicted rates for men and women together using matplot:

```
> matplot( nd$A, cbind(p.m,p.f),
+ type="l", col=rep(c("blue","red"),each=3), lwd=c(3,1,1), lty=1,
+ log="y", xlab="Age", ylab="Mortality of DM ptt per 1000 PY")
```

Period and duration effects

8. We model the mortality rates among diabetes patients also including current date and duration of diabetes. However, we shall not just use the positioning of knots for the splines as provided by ns, because this is based on the allocating knots so that the number of observations (lines in the dataset), is the same between knots.

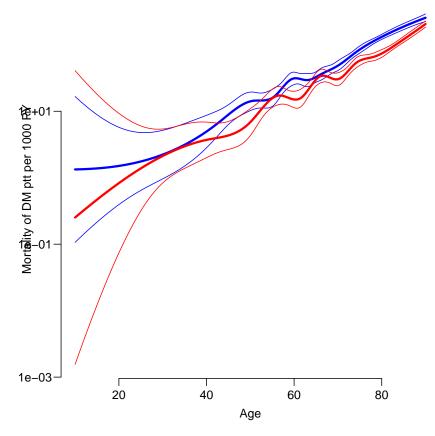


Figure 3.4: Age-specific mortality rates for Danish diabetes patients as estimated from a model with only age. Blue: men, red: women.

However the information in a follow-up study is in the number of events, so it would be better to allocate knots so that number of events were the same between knots.

We use the so-called *natural splines* that are linear beyond the boundary knots, and hence we take the 5th and 95th percentile of deaths as the boundary knots for age (A) and calendar time (P) but for duration where we actually have follow-up from tine 0 on the timescale we use 0 as the first knot.

```
( kn.A <- with( subset( SL, lex.Xst=="Dead" ),
                  quantile( A+lex.dur, probs=seq(5,95,20)/100 ) )
      5%
              25%
                        45%
                                 65%
56.02519 69.06092 76.29021 81.42094 87.66598
> ( kn.P <- with( subset( SL, lex.Xst=="Dead" ),</pre>
                  quantile( P+lex.dur, probs=seq(5,95,20)/100 ) )
                        45%
                                 65%
1998.117 2002.120 2004.694 2006.826 2008.761
  ( kn.dur <- c(0, with( subset( SL, lex.Xst=="Dead" ),
                  quantile( dur+lex.dur, probs=seq(10,90,20)/100 ) )) )
                                     50%
                                               70%
                                                          90%
0.0000000 0.3055441 1.5961670 3.4250513 5.6629706 9.1723477
```

9. With these we can now model mortality rates (separately for men and women), as functions of age, calendar time and duration:

```
> Mm <- glm( (lex.Xst=="Dead") ~ Ns( A, kn=kn.A ) +
                                Ns(P, kn=kn.P) +
                                Ns( dur, kn=kn.dur ),
            offset = log( lex.dur ),
            family = poisson,
              data = subset( SL, sex=="M" ) )
> summary( Mm )
glm(formula = (lex.Xst == "Dead") ~ Ns(A, kn = kn.A) + Ns(P,
    kn = kn.P) + Ns(dur, kn = kn.dur), family = poisson, data = subset(SL,
    sex == "M"), offset = log(lex.dur))
Deviance Residuals:
   Min
             1Q
                 Median
                               3Q
                                       Max
-0.8367 -0.2308 -0.1595 -0.1115
                                    4.4965
Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)
                     -3.16121 0.10390 -30.426 < 2e-16
Ns(A, kn = kn.A)1
                     1.52180
                                 0.11720 12.985 < 2e-16
Ns(A, kn = kn.A)2
                                 0.09175
                     1.89400
                                         20.643 < 2e-16
Ns(A, kn = kn.A)3
                     2.98735
                                0.12279
                                         24.328 < 2e-16
Ns(A, kn = kn.A)4
                     2.05374
                                 0.07824
                                         26.250 < 2e-16
Ns(P, kn = kn.P)1
                                 0.13352
                                         -1.461 0.144009
                     -0.19507
Ns(P, kn = kn.P)2
                     -0.29731
                                         -2.780 0.005435
                                 0.10694
Ns(P, kn = kn.P)3
                     -0.43455
                                 0.17152
                                         -2.533 0.011293
Ns(P, kn = kn.P)4
                                         -3.295 0.000982
                     -0.29586
                                 0.08978
Ns(dur, kn = kn.dur)1 - 0.76626
                                 0.15497
                                         -4.945 7.63e-07
                                         -4.124 3.72e-05
Ns(dur, kn = kn.dur)2 -0.63208
                                 0.15325
Ns(dur, kn = kn.dur)3 -0.46099
                                0.12080
                                         -3.816 0.000136
Ns(dur, kn = kn.dur)4 -1.29240
                                0.21518
                                         -6.006 1.90e-09
Ns(dur, kn = kn.dur)5 -0.12241
                               0.09654
                                         -1.268 0.204796
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 12999 on 60346 degrees of freedom
Residual deviance: 11727 on 60333 degrees of freedom
AIC: 14441
Number of Fisher Scoring iterations: 7
> Mf <- update( Mm, data = subset( SL, sex=="F" ) )</pre>
> round( cbind( ci.exp(Mm), ci.exp(Mf) ), 3 )
                     exp(Est.)
                                 2.5% 97.5% exp(Est.)
                                                        2.5% 97.5%
(Intercept)
                         0.042 0.035 0.052 0.025 0.019 0.032
                                               4.247 3.199 5.639
Ns(A, kn = kn.A)1
                         4.580 3.640 5.763
Ns(A, kn = kn.A)2
                        6.646 5.552 7.955
                                               5.079 4.180 6.170
Ns(A, kn = kn.A)3
                        19.833 15.591 25.230
                                               20.611 15.288 27.788
Ns(A, kn = kn.A)4
                         7.797
                               6.689 9.089
                                               7.806
                                                       6.572 9.272
Ns(P, kn = kn.P)1
                               0.633 1.069
                                                0.908
                                                       0.686
                                                              1.202
                        0.823
Ns(P, kn = kn.P)2
                        0.743
                               0.602 0.916
                                                0.730
                                                       0.579 0.921
Ns(P, kn = kn.P)3
                        0.648 0.463 0.906
                                                0.768
                                                       0.524
                                                             1.125
Ns(P, kn = kn.P)4
                        0.744 0.624 0.887
                                                0.668 0.551 0.809
                                             0.541 0.387 0.756
Ns(dur, kn = kn.dur)1 0.465 0.343 0.630
```

```
Ns(dur, kn = kn.dur)2
                           0.531
                                  0.394
                                          0.718
                                                    0.472
                                                            0.338
                                                                   0.658
Ns(dur, kn = kn.dur)3
                           0.631
                                  0.498
                                          0.799
                                                    0.871
                                                            0.678
                                                                   1.118
Ns(dur, kn = kn.dur)4
                           0.275
                                  0.180
                                                    0.398
                                                            0.248
                                                                   0.641
                                          0.419
Ns(dur, kn = kn.dur)5
                           0.885
                                  0.732
                                          1.069
                                                    0.982
                                                            0.800
```

It is not possible to attach any meaning to the single parameters from the model, so we shall look at the estimated non-linear effects of each of the variables.

10. These models fit substantially better than the model with only age as we can see from this comparison:

```
> anova( Mm, r.m, test="Chisq" )
Analysis of Deviance Table
Model 1: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A) + Ns(P, kn = kn.P) + Ns(dur,
    kn = kn.dur)
Model 2: (lex.Xst == "Dead") \sim ns(A, df = 10)
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
      60333
                 11727
2
      60336
                 11808 -3 -81.122 < 2.2e-16
> anova( Mf, r.f, test="Chisq" )
Analysis of Deviance Table
Model 1: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A) + Ns(P, kn = kn.P) + Ns(dur,
    kn = kn.dur)
Model 2: (lex.Xst == "Dead") \sim ns(A, df = 10)
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
      58112
                 10203
2
      58115
                 10258 -3
                           -54.302 9.675e-12
```

The models are not formally nested since the location of the age-knots is different, so from a formal point of view these test are not valid, but is is clear that the more extensive modeling provides a much better description of the rates.

11. We can inspect the shape of the estimated effects (and their relative size) using the Termplot function in the Epi package.

However in order for this to work properly we need a model specification where *all* of the prediction is part of a term, essentially including the intercept in one of the terms — notably age. Moreover, the age-specific rates must the refer to a specific period and diabetes duration.

This is done by using the intercept and ref arguments to Ns:

We can check that it actually is the same model, by using the deviances from the two models fitted.

```
> c( deviance(Mm), deviance(mm) )
[1] 11726.61 11726.61
```

12. We the use Termplot, which is a wrapper for termplot. Termplot gives plots on the rate / resp RR scale, so that we can actually make sense of the plots.

```
> Termplot( mm )
```

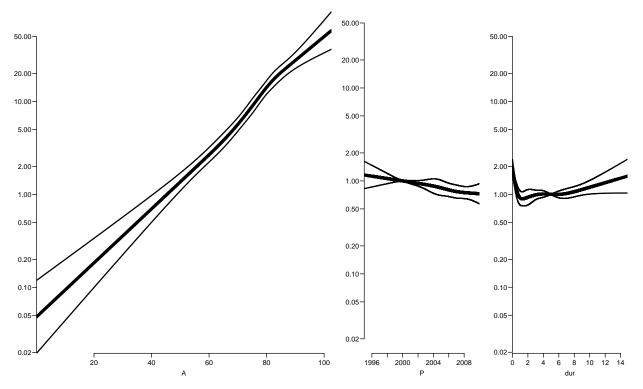


Figure 3.5: Age, period and duration terms for mortality among Danish male diabetes patients. The age effect is age-specific rates for persons with 5 years of diabetes duration in the year 2000.

```
> Termplot( mf )
```

13. Since the fitted model has three time-scales: current age, current date and current duration of diabetes, so the effects that we see in the Termplot are not really interpretable; they are (as in any kind of multiple regressions) to be interpreted as "all else equal" which they are not; the three time scales advance simultaneously at the same pace.

The reporting would therefore more naturally be *only* on one time scale, showing the mortality for persons diagnosed in different ages in a given year.

This is most easily done using the ci.pred function with the newdata= argument. So a person diagnosed in age 50 in 1995 will have a mortality measured in cases per 1000 PY as:

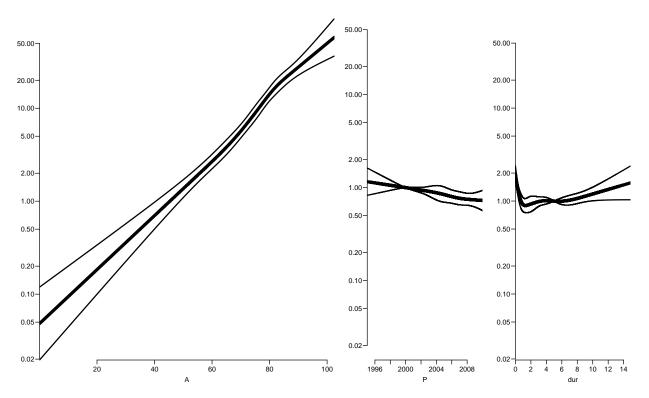


Figure 3.6: Age, period and duration terms for mortality among Danish female diabetes patients. The age effect is age-specific rates for persons with 5 years of diabetes duration in the year 2000.

```
> pts <- seq(0,20,2)
 nd <- data.frame( A= 50+pts,
                     P=1995+pts,
                            pts,
                   dur=
              lex.dur=1000 )
  cbind( nd$A, ci.pred( mm, newdata=nd ) )
      Estimate
                    2.5%
                             97.5%
1
   50
     31.02982 21.72823
                          44.31332
      15.85329 12.17126
                          20.64919
3
      18.54048 15.12524
                          22.72687
      19.75660 16.17241
                          24.13514
5
   58
      22.71326 18.97210
                          27.19216
6
   60
     26.79926
               22.01942
                          32.61667
                          40.65463
7
   62
      31.43090
               24.29985
8
   64 38.99649
               27.98085
                          54.34883
                          80.70802
   66 49.17746 29.96508
10 68 62.48983 31.13183 125.43363
11 70 80.12938 32.18765 199.47763
```

Since there is no duration beyond 18 years in the dataset we only make predictions for 20 years of duration, and do it for persons diagnosed in 1995 and 2005 — the latter is quite dubious too because we are extrapolating calendar time trends way beyond data.

We form matrices of predictions, that we will plot in the same frame:

```
> mpr <- fpr <- NULL
> pts <- seq(0,20,0.1)
> for(ip in c(1995, 2005))
+ for( ia in c(50,60,70) )
+ nd <- data.frame( A=ia+pts,
                    P=ip+pts,
                  dur= pts,
             lex.dur=1000 )
+ mpr <- cbind( mpr, ci.pred( mm, nd) )
+ fpr <- cbind( fpr, ci.pred( mf, nd) )
  7
> str( fpr )
num [1:201, 1:18] 14.5 13.1 12 11 10.3 ...
  attr(*, "dimnames")=List of 2
  ..$ : chr [1:201] "1" "2" "3" "4"
  ..$ : chr [1:18] "Estimate" "2.5%" "97.5%" "Estimate" ...
```

These 18 columns are 9 columns for 1995, and 9 for 2005, each of these chunks are estimate and lower and upper confidence bound for persons diagnosed in ages 50, 60 and 70.

These can now be plotted:

3.2.1 SMR.

There are two ways to make the comparison of the diabetes mortality to the population mortality; one is to amend the diabetes patient dataset with the population mortality dataset, the other (classical) one is to include the population mortality rates as a fixed variable in the calculations.

The latter requires that each analytic unit in the diabetes patient dataset is amended with a variable with the population mortality rate for the corresponding sex, age and calendar time.

This can be achieved in two ways: Either we just use the current split of follow-up time and allocate the population mortality rates for some suitably chosen (mid-)point of the follow-up in each, or we make a second split by date, so that follow-up in the diabetes patients is in the same classification of age and data as the population mortality table.

14. Using the former approach we shall include as an extra variable the population mortality as available from the data set M.dk.

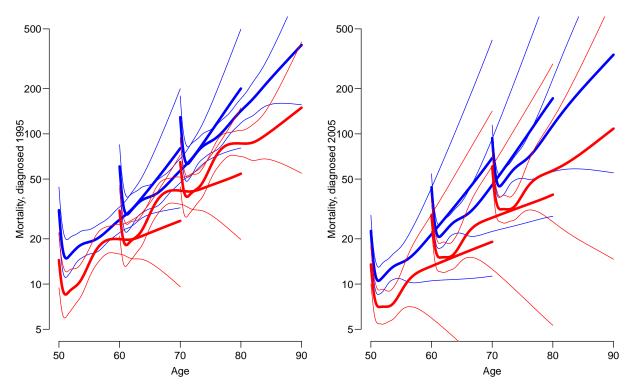


Figure 3.7: Mortality rates for diabetes patients diagnosed 1995 and 2005 in ages 50, 60 and 70. Men blue, women red.

First create the variables in the diabetes dataset that we need for matching with the age and period classification of the population mortality data, that is age, date (and sex) at the midpoint of each of the intervals (or rater at a point 3 months after the left endpoint of the interval — recall we split the follow-up in 6 month intervals).

We need to have variables of the same type when we merge, so we must transform the sex variable in M.dk to a factor, and must for each follow-up interval in the SL data have an age and a period variable that can be used in merging with the population data.

> str(SL)

```
Classes 'Lexis'
                and 'data.frame':
                                         118473 obs. of
                                                        14 variables:
 $ lex.id : int
                 1 1 1 1 1 1 1 1 1 1
 $
                 58.7 59 59.5 60 60.5 ...
          : num
 $
  Ρ
                 1999 1999 2000 2000 2001
          : num
 $
          : num
                 0 0.339 0.839 1.339 1.839 ...
  lex.dur: num
                 0.339 0.5 0.5 0.5 0.5 ...
  lex.Cst: Factor w/ 2 levels "Alive", "Dead": 1 1 1 1 1 1 1 1 1 1 1 ...
  lex.Xst: Factor w/ 2 levels "Alive", "Dead": 1 1 1 1 1 1 1 1 1 1 1 ...
          : Factor w/ 2 levels "M", "F": 2 2 2 2 2 2 2 2 2 2 ...
 $
  sex
                 1940 1940 1940 1940 ...
  dobth
           num
 $
  dodm
           num
                 1999 1999 1999 1999
  dodth
                 NA NA NA NA NA NA NA NA
           num
                 NA NA NA NA NA NA NA NA NA
  dooad
           num
 $
  doins
                 NA NA NA NA NA NA NA NA NA
  dox
                 2010 2010 2010 2010 2010 ...
   attr(*, "breaks")=List of 3
```

```
: num 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 ...
  ..$ P
        : NULL
  ..$ dur: NULL
 - attr(*, "time.scales")= chr "A" "P" "dur"
 - attr(*, "time.since")= chr
                              0.00 0.00 0.00
> SL$Am <- floor( SL$A+0.25 )
> SL$Pm <- floor( SL$P+0.25 )
> data( M.dk )
> str( M.dk )
'data.frame':
                    7800 obs. of 6 variables:
$ A : num 0 0 0 0 0 0 0 0 0 ...
$ sex : num 1 2 1 2 1 2 1 2 1 2 ...
      : num 1974 1974 1975 1975 1976 ...
       : num 459 303 435 311 405 258 332 205 312 233 ...
$ D
$ Y
             35963 34383 36099 34652 34965 ...
      : num
$ rate: num 12.76 8.81 12.05 8.97 11.58 ...
  attr(*, "Contents") = chr "Number of deaths and risk time in Denmark"
> M.dk <- transform( M.dk, Am = A,
                          sex = factor( sex, labels=c("M", "F") ) )
> str( M.dk )
'data.frame':
                    7800 obs. of 8 variables:
     : num 0000000000...
 $ sex : Factor w/ 2 levels "M". "F": 1 2 1 2 1 2 1 2 1 2 ...
      : num
             1974 1974 1975 1975 1976 . . .
      : num 459 303 435 311 405 258 332 205 312 233 ...
$ D
      : num 35963 34383 36099 34652 34965 ...
             12.76 8.81 12.05 8.97 11.58 ...
$ rate: num
  Am : num 0 0 0 0 0 0 0 0 0 ...
             1974 1974 1975 1975 1976
  Pm
      : num
```

We then match the rates from M.dk into SL — sex, Am and Pm are the common variables, and therefore the match is on these variables:

This merge only takes rows that have information from both data sets, hence the slightly fewer rows in SLr than in SL — there are a few record in SL with age and period values that do not exist in the population mortality data.

15. We compute the expected number of deaths as the person-time multiplied by the corresponding population rate recalling that the rate is given in units of deaths per 1000 PY, whereas lex.dur is in units of 1 PY:

```
> SLr$E <- SLr$lex.dur * SLr$rate / 1000
> stat.table( sex,
            list( D = sum(lex.Xst=="Dead"),
                 Y = sum(lex.dur),
                 E = sum(E),
                SMR = ratio(lex.Xst=="Dead",E) ),
            data = SLr)
          D Y E SMR
     1342.00 27611.40 796.11 1.69
     1153.00 26654.52 747.77 1.54
> stat.table( list( Age = floor(pmax(A,39)/10)*10 ),
            list(D = sum(lex.Xst == "Dead"),
                 Y = sum(lex.dur),
                 E = sum(E),
                SMR = ratio(lex.Xst=="Dead",E) ),
            data = SLr)
          D Y E
                               SMR
      11.00 4706.00 3.18 3.45
       47.00 5776.18 14.48 3.25
40
      181.00 10765.19 70.47
                              2.57
50
      432.00 14052.52 216.39
60
                             2.00
      817.00 12225.99 480.11
70
                              1.70
       771.00 5952.59 573.73
                              1.34
80
90
       236.00 787.46 185.51
```

We see that the SMR is slightly higher for women than for men, but also that there is a much larger variation in SMR by age.

16. We can the SMR exactly as mortality rates by including the log expected numbers instead of the log person-years as offset, again using separate models for men and women.

We exclude those records where no deaths in the population occur (that is where the rate is 0) — you could say that this correspond to parts of the data where no follow-up on the population mortality scale is available.

We can plot the estimates as before for the rates, using Termplot. What do the extracted effects represent now?

```
> Termplot( sm )
```

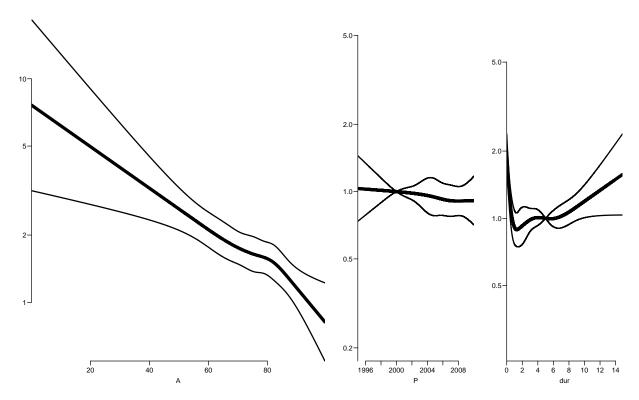


Figure 3.8: Age, period and duration terms for SMR among Danish male diabetes patients. The age effect is age-specific SMR for persons with 5 years of diabetes duration in the year 2000.

> Termplot(sf)

17. We can check if there are different SMRs between men and women by fitting a joint model and expanding it with (linear) sex-effect(s):

```
> s0 <- glm( (lex.Xst=="Dead") ~ Ns(
                                          A, kn=kn.A , intercept=TRUE ) - 1 +
                                          P, kn=kn.P , ref=2000 ) +
                                    Ns(
                                    Ns( dur, kn=kn.dur, ref=5 ),
              offset = log(E),
              family = poisson,
                data = subset( SLr, E>0 ) )
> s1 <- update( s0, . ~ . + sex )
> sA <- update( s1, . ~ . + sex:A
                          . + sex:A )
> sAP <- update( sA, . ~ . + sex:A )
> sAPd <- update( sAP, . ~ . + sex:d
                             . + sex:dur )
> anova( s0, s1, sA, sAP, sAPd, test="Chisq" )
Analysis of Deviance Table
Model 1: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) - 1 +
    Ns(P, kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5)
Model 2: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex - 1
Model 3: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A - 1
```

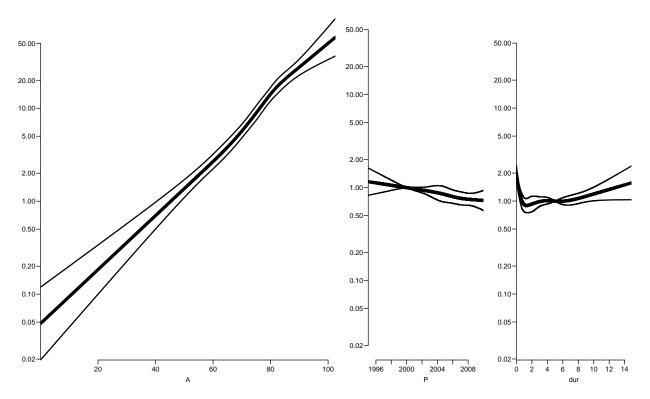


Figure 3.9: Age, period and duration terms for mortality among Danish female diabetes patients. The age effect is age-specific SMR for persons with 5 years of diabetes duration in the year 2000.

```
Model 4: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A + sex:P - 1
Model 5: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A + sex:P + sex:dur - 1
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
                 21925
1
     118418
2
     118417
                            0.00799
                                      0.9288
                 21925
                        1
3
     118416
                 21925
                        1
                            0.12948
                                      0.7190
4
     118415
                 21925
                        1
                            0.23766
                                      0.6259
     118414
                 21924
                            0.45765
                                      0.4987
```

So by this simple check we see there is no really compelling evidence that the SMR differs between men and women.

Of course we might repeat it all by including quadratic effects too:

```
> sA <- update( s1, . ~ . + sex:A + sex:I(A^2) )
> sAP <- update( sA, . ~ . + sex:P + sex:I(P^2) )
> sAPd <- update( sAP, . ~ . + sex:dur + sex:I(dur^2) )
> anova( s0, s1, sA, sAP, sAPd, test="Chisq" )
Analysis of Deviance Table

Model 1: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) - 1 +
        Ns(P, kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5)
Model 2: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
```

```
kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
Model 3: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A + sex:I(A^2) - 1
Model 4: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A + sex:I(A^2) + sex:P + sex:I(P^2) - 1
Model 5: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) + Ns(P,
    kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5) +
    sex + sex:A + sex:I(A^2) + sex:P + sex:I(P^2) + sex:dur +
    sex:I(dur^2) - 1
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1
     118418
                 21925
2
     118417
                 21925
                        1
                             0.0080
                                      0.9288
3
     118414
                 21923
                        3
                             2.4179
                                      0.4903
4
     118411
                 21919
                        3
                             3.8787
                                      0.2749
     118408
                                      0.7354
                 21918
                             1.2737
```

So there really is no difference, so we can report the SMR between the diabetes patients and the population as sex-independent:

> Termplot(s0)

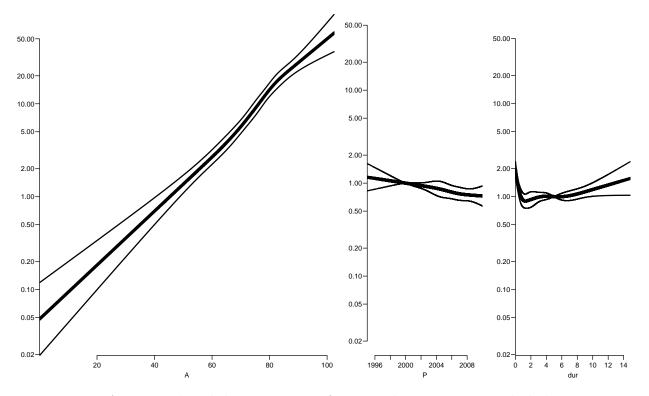


Figure 3.10: Age, period and duration terms for mortality among Danish diabetes patients, male as well as female. The age effect is age-specific SMR for persons with 5 years of diabetes duration in the year 2000.

18. As before, it would be more sensible to show the SMR as a function of age for persons diagnosed with DM at ages 50, 60 and 70. The code is essentially the same as before:

These 18 columns are 9 columns for 1995, and 9 for 2005, each of these chunks are estimate and lower and upper confidence bound for persons diagnosed in ages 50, 60 and 70.

These can now be plotted:

From the figure it seems that the conclusion is that there is no effect of age or *current* age on SMR, but pretty much that there is an effect of age at diagnosis and a very strong initial effect of diabetes duration.

19. Try to simplify the model to one with a simple linear effect of date of diagnosis, and using only knots at 0,1,and 2 years for duration, giving an estimate of the change in SMR as duration increases beyond 2 years.

It would be natural to simplify the model to one with a non-linear effect of duration and linear effects of age at diagnosis and calendar time. We choose knots with successive distances of 1,2,3 and 4 years (a bit out of the blue):

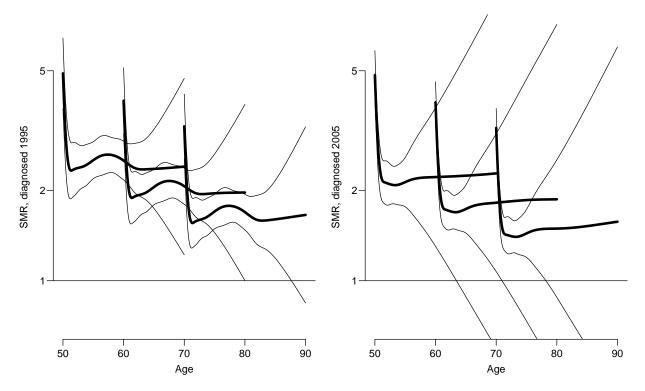


Figure 3.11: SMR rates for diabetes patients diagnosed 1995 and 2005 in ages 50, 60 and 70.

```
Analysis of Deviance Table

Model 1: (lex.Xst == "Dead") ~ Ns(A, kn = kn.A, intercept = TRUE) - 1 +
    Ns(P, kn = kn.P, ref = 2000) + Ns(dur, kn = kn.dur, ref = 5)

Model 2: (lex.Xst == "Dead") ~ I(A - dur) + I(P - 2000) + Ns(dur, kn = c(0,
    1, 3, 6, 10), ref = 5)
    Resid. Df Resid. Dev Df Deviance Pr(>Chi)

1 118418 21925
2 118425 21935 -7 -9.9485 0.1915
```

Thus there is no difference between the very simple model for SMR and the more complicated ones; and we see that the change in SMR per year of age at diagnosis and calendar year is pretty much the same, namely some 2% per year, or some $15{\text -}18\%$ per 10 years:

```
> round( ci.exp(sx), 4 )
                                           exp(Est.)
                                                       2.5%
                                                             97.5%
(Intercept)
                                              6.7442 5.1886 8.7660
I(A - dur)
                                              0.9809 0.9775 0.9843
I(P - 2000)
                                              0.9843 0.9726 0.9962
Ns(dur, kn = c(0, 1, 3, 6, 10), ref = 5)1
                                              0.5777 0.4830 0.6908
Ns(dur, kn = c(0, 1, 3, 6, 10), ref = 5)2
                                              0.7085 0.6036 0.8316
Ns(dur, kn = c(0, 1, 3, 6, 10), ref = 5)3
                                              0.2638 0.1956 0.3560
Ns(dur, kn = c(0, 1, 3, 6, 10), ref = 5)4
                                              0.9187 0.8040 1.0497
> round( ci.exp( sx, subset=c("A", "P"), ctr.mat=10*diag(2) ), 4 )
```

```
exp(Est.) 2.5% 97.5% [1,] 0.8247 0.7966 0.8537 [2,] 0.8536 0.7572 0.9623
```

20. We can also see that the predicted SMRs looks pretty much the same:

```
> xsmr <- NULL
> for(ip in c(1995, 2005))
 for ( ia in c(50,60,70) )
 nd <- data.frame( A=ia+pts,</pre>
                    P=ip+pts,
                  dur=
                         pts,
                    E=1)
 xsmr <- cbind( xsmr, ci.pred( sx, nd) )</pre>
 par( mfrow=c(1,2) )
 matplot( cbind(50+pts,60+pts,70+pts)[,rep(1:3,each=3)],
           xsmr[,1:9], ylim=c(0.7,7),
           log="y", xlab="Age", ylab="SMR, diagnosed 1995",
           type="1", lwd=c(4,1,1), lty=1, col="black")
 abline( h=1 )
 matplot( cbind(50+pts,60+pts,70+pts)[,rep(1:3,each=3)],
           xsmr[,1:9+9], ylim=c(0.7,7),
           log="y", xlab="Age", ylab="SMR, diagnosed 2005",
           type="1", lwd=c(4,1,1), lty=1, col="black")
> abline( h=1 )
```

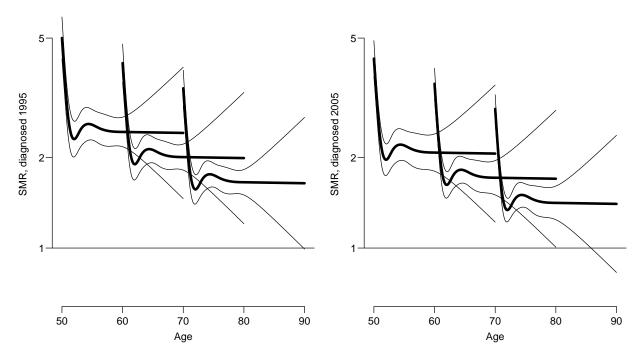


Figure 3.12: SMR rates for diabetes patients diagnosed 1995 and 2005 in ages 50, 60 and 70. Simplified model.

From the figure it seems that the conclusion is that there is no effect of *current* age on SMR, but pretty much that there is an effect of age at diagnosis and a very strong initial effect of diabetes duration.