# Multistate models in Epi 

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## 1 Introduction

The following exposition of multistate manipulations follows (parts of) the exposition by Hein Putter which in turn is a computational companion for R to the tutorial by Putter et al. [?] from Stat. Med. 2007.

## 2 The example data

The dataset used is about the clinical course of bone-marrow transplant patients. The major event of interest is relapse or death (RD), and an intermediate event is platelet recovery (PR).

We first show the current version and then load the Epi and the mstate packages:

```
> options( "width" )
$width
[1] }8
> options( width=120 )
> R.version[c("platform","version.string")]
platform i}3386-pc-mingw3
version.string R version 2.9.2 (2009-08-24)
> installed.packages()[c("Epi","mstate"),c("Version","Built")]
ll
> library(Epi)
> library(mstate)
```

From the mstate package we get the ebmt3 dataset:

```
> data( ebmt3 )
> str( ebmt3 )
'data.frame': 2204 obs. of 9 variables:
    $ id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ prtime : num 23 35 26 22 29 ...
    $ prstat : int 1 1 1 1 1 1 1 1 0 0 .. 
    $ rfstime: num 744 360 135 995 422 ...
    $ rfsstat: int 0 1 1 0 1 1 0 0 0 1 ...
    $ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3...
    $ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
    $ drmatch: Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 1 2 1 1 1 ...
    $ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
> head( ebmt3 )
```

| 1 | d |  |  | e |  | ub | age |  |  | drmatch | cd |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 23 | 1 | 744 | 0 | CML | >40 |  | Gender | mismatch N | No TCD |
| 2 | 2 | 35 | 1 | 360 | 1 | CML | >40 | No | gender | mismatch N | No TCD |
| 3 | 3 | 26 | 1 | 135 | 1 | CML | >40 | No | gender | mismatch N | No TCD |
| 4 | 4 | 22 | 1 | 995 | 0 | AML | 20-40 | No | gender | mismatch N | No TCD |
| 5 | 5 | 29 | 1 | 422 | 1 | AML | 20-40 | No | gender | mismatch N | No TCD |
| 6 | 6 | 38 | 1 | 119 | 1 | ALL | >40 | No | gender | mismatch N | No TCD |

The times are in days since transplant (Tx), they are time to relapse or death (RD) in the variable rfstime, and time to platelet recovery (PR) in the variable prtime. PR is considered an intermediate state.

Some of the platelet recovery times are larger than the recurrence times; these are (supposedly) instances where platelet recovery occurs after a relapse. However death and relapse are not distinguished in the available dataset.

```
> with(ebmt3,plot(prtime,rfstime,pch=16,col=c("gray","red")[prstat+1]))
```



Figure 1: The two time variables, color coded by the value of prstat, 0 is gray, 1 is red.

## 3 A Lexis object

A convenient way of storing the information on the multistate structure is a Lexis object as supported by the Epi package. A Lexis object is basically a data frame with some extra
columns added. Each row represents a piece of follow-up, with indication of starting point on the timescale(s) and length of follow-up. The former are variables for each timescale, the latter (which is common on all time scales) in the variable (lex.dur (duration). The state in which the follow-up takes place is called lex.Cst (Current state) and the state occupied immediately after the follow-up lex.Xst (Xit state). This type of object is constructed using the Lexis function.

The entry and exit times are given as named lists with times on the timescales of interest. For one of these, only one timescale need be specified, since the duration (length of follow-up) is the same on all timescales. One of these can optionally be replaced by duration.

In this study everyone enters at time 0 , so we need not specify the argument entry to the Lexis function (it is by default set to 0 of only either exit or duration is given).
Moreover, since everyone starts in state Tx , we just need to specify this state as the first level of the state factor, in order to have everyone assigned this as entry state by default:

```
> bmt <- Lexis( exit = list(tft=rfstime/365.25),
+ exit.status = factor(rfsstat,labels=c("Tx","RD")),
+ data = ebmt3 )
NOTE: entry.status has been set to "Tx" for all.
NOTE: entry is assumed to be 0 on the tft timescale.
```

```
> str( bmt )
```

> str( bmt )
Classes 'Lexis' and 'data.frame': 2204 obs. of 14 variables:
Classes 'Lexis' and 'data.frame': 2204 obs. of 14 variables:
\$ tft : num 0 0 0 0 0 0 0 0 0 0 ...
\$ tft : num 0 0 0 0 0 0 0 0 0 0 ...
\$ lex.dur: num 2.037 0.986 0.37 2.724 1.155 ...
\$ lex.dur: num 2.037 0.986 0.37 2.724 1.155 ...
\$ lex.Cst: Factor w/ 2 levels "Tx","RD": 1 1 1 1 1 1 1 1 1 1 ...
\$ lex.Cst: Factor w/ 2 levels "Tx","RD": 1 1 1 1 1 1 1 1 1 1 ...
\$ lex.Xst: Factor w/ 2 levels "Tx","RD": 1 2 2 1 2 2 1 1 1 2 ...
\$ lex.Xst: Factor w/ 2 levels "Tx","RD": 1 2 2 1 2 2 1 1 1 2 ...
\$ lex.id : int 1 2 3 4 5 6 7 8 9 10 ...
\$ lex.id : int 1 2 3 4 5 6 7 8 9 10 ...
\$ id : int 1 2 3 4 5 6 7 8 9 10 ...
\$ id : int 1 2 3 4 5 6 7 8 9 10 ...
\$ prtime : num 23 35 26 22 29 ...
\$ prtime : num 23 35 26 22 29 ...
\$ prstat : int 1 1 1 1 1 1 1 1 0 0 0 ..
\$ prstat : int 1 1 1 1 1 1 1 1 0 0 0 ..
\$ rfstime: num 744 360 135 995 422 ...
\$ rfstime: num 744 360 135 995 422 ...
\$ rfsstat: int 011011 1 0 0 0 1 ...
\$ rfsstat: int 011011 1 0 0 0 1 ...
\$ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3 ...
\$ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3 ...
\$ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
\$ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
\$ drmatch: Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 2 1 1 1 ...
\$ drmatch: Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 2 1 1 1 ...
\$ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
\$ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
- attr(*, "time.scales")= chr "tft"
- attr(*, "time.scales")= chr "tft"
- attr(*, "breaks")=List of 1
- attr(*, "breaks")=List of 1
..\$ tft: NULL

```
        ..$ tft: NULL
```

The time scaling was redefined to years on the fly, but we only defined the two-state model with transition from Tx to RD. This simple structure is shown by the summary command:

```
> summary( bmt )
Transitions:
        To
From Tx RD Records: Events: Risk time: Persons:
Rates:
            To
From Tx RD Total
    Tx 0}00.15 0.1
```

Thus we have 841 events during 5612 person-years, corresponding to a rate of 0.15 events per year.

We have only one timescale in operation, so the default plot is a so-called 1-dimensional Lexis diagram:

```
> par(mar=c(3,3,1,1), mgp=c(3,1,0)/1.6 )
> plot( bmt )
```



Figure 2: One-dimensional Lexis diagram of the bone-marrow transplant data from ebmt3.
Now we want to introduce the intermediate event platelet recovery, PR. We have the date of recovery in the variable prtime. Since all the censored values for prtime are after the relapse/death time, we could just use this variable. However, for the sake of the argument we first set the censored values of prtime to NA:

```
> bmt$prtime[bmt$prstat==0] <- NA
```

Now we can introduce the new state, and subdivide the follow-up time of the patients according to platelet recovery. We must of course specify the name of the new state in which patients enter at prtime, but also the so called precursor states, that is states that are overridden by the new state. This is only Tx because the RD state is absorbing. In practice it means that if a person exits the study in state Tx (any precursor state), then an entry to the new state (PR) will mean that the person exits in the new state, whereas if the person exits the study to state RD (any non-precursor state) the exit from the study will still be to this (original) state, regardless of whether the intermediate event has occurred or not:

```
> bmtr <- cutLexis( bmt, cut=bmt$prtime/365.25,
+ pre="Tx",
+ new.state="PR" )
> str( bmtr )
Classes 'Lexis' and 'data.frame': 3373 obs. of 14 variables:
    $ tft : num 0 0 0 0 0 0 0 0 0 0 ...
    $ lex.dur: num 0.063 0.0958 0.0712 0.0602 0.0794 ...
    $ lex.Cst: Factor w/ 3 levels "Tx","PR","RD": 1 1 1 1 1 1 1 1 1 1 ...
    $ lex.Xst: Factor w/ 3 levels "Tx","PR","RD": 2 2 2 2 2 2 2 2 1 3...
    $ lex.id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ prtime : num 23 35 26 22 29 38 30 35 NA NA ...
    $ prstat : int 1 1 1111111111 0 0 ...
    $ rfstime: num 744 360 135 995 422 ...
    $ rfsstat: int 0 1 1 0 1 1 0 0 0 1 ...
    $ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3 ...
    $ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
    $ drmatch: Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 2 1 1 1 ...
    $ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
    - attr(*, "time.scales")= chr "tft"
    - attr(*, "breaks")=List of 1
        ..$ tft: NULL
> summary( bmtr )
Transitions:
            To
\begin{tabular}{crrrrrr} 
From & Tx & PR & RD & Records: & Events: & Risk time:
\end{tabular} Persons:
Rates:
            To
From Tx PR RD Total
    Tx 0 0.48 0.19 0.67
    PR 0 0.00 0.12 0.12
```

We see that we now have 3373 records instead of 2204 , because of the 1169 transitions to PR. We still have the same amount of risk time (5612 person-years) and deaths (841) in total.

By default the newly created state is placed after the precursor states nominated, and before all non-procursor states.

### 3.1 Clock-back approach

We may be interested in the "clock back" approach, that is using the duration of platelet recovery as timescale. This can be accomplished by the new.scale argument to cutLexis. If set to TRUE a new timescale will generated, named PR.dur; alternatively we can set the argument to the name of the new timescale:

```
> bmtr <- cutLexis( bmt, cut=bmt$prtime/365.25,
+ pre="Tx",
+ new.state="PR", new.scale="tfPR" )
> str( bmtr )
```

```
Classes 'Lexis' and 'data.frame': 3373 obs. of 15 variables:
    $ tft : num 0 0.063 0 0.0958 0 ...
    $ tfPR : num NA 0 NA 0 NA 0 NA 0 NA 0 ...
    $ lex.dur: num 0.063 1.974 0.0958 0.8898 0.0712 ...
    $ lex.Cst: Factor w/ 3 levels "Tx","PR","RD": 1 2 1 2 1 2 1 2 1 2 ...
    $ lex.Xst: Factor w/ 3 levels "Tx","PR","RD": 2 2 2 3 2 3 2 2 2 3 ...
    $ lex.id : int 1 1 2 2 3 3 4 4 5 5 ...
    $ id : int 1 1 2 2 3 3 4 4 5 5 ...
    $ prtime : num 23 23 35 35 26 26 22 22 29 29 ...
    $ prstat : int 1 1 1 1 1 1 1 1 1 1 1 1 1...
    $ rfstime: num 744 744 360 360 135 135 995 995 422 422 ...
    $ rfsstat: int 0 0 1 1 1 1 0 0 1 1 ...
    $ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 3 3 3 1 1 1 1 ...
    $ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 3 3 3 2 2 2 2 ...
    $ drmatch: Factor w/ 2 levels "No gender mismatch",..: 2 2 1 1 1 1 1 1 1 1 ...
    $ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
    - attr(*, "time.scales")= chr "tft" "tfPR"
    - attr(*, "breaks")=List of 2
    ..$ tft : NULL
    ..$ tfPR: NULL
> summary( bmtr )
Transitions:
            To
\begin{tabular}{crrrrrrr} 
From & Tx & PR & RD & Records: & Events: & Risk time: & Persons: \\
Tx & 577 & 1169 & 458 & 2204 & 1627 & 2439.43 & 2204 \\
PR & 0 & 786 & 383 & 1169 & 383 & 3173.03 & 1169 \\
Sum 577 & 1955 & 841 & 3373 & 2010 & 5612.46 & 2204
\end{tabular}
Rates:
        To
From Tx PR RD Total
    Tx 0 0.48 0.19 0.67
    PR 0 0.00 0.12 0.12
```

It is seen that this representation retains the total amount of risk time in the study; it merely subdivides it by state. It is seen that the output of the summary. Lexis function is pretty similar to the output from the events function from the mstate package.

Moreover, since we now have two timescales we can make a two-dimensional Lexis diagram, as shown in figure 1. The diagram only shows follow-up after PR, since one of the timescales is time since PR, which obviously is not defined before the event.

```
> par( mar=c(3,3,1,1), mgp=c(3,1,0)/1.6 )
> plot( bmtr )
```


### 3.2 Subdividing states

We may also be interested in subdividing the states subsequent to PR according to whether this event has occurred or not. In this example only RD is subsequent to the event PR:

```
> bmtr <- cutLexis( bmt, cut=bmt$prtime/365.25,
+ pre="Tx",
+ new.state="PR", new.scale="tfPR",
+ split.states=TRUE )
> summary( bmtr )
```



Figure 3: Two-dimensional Lexis diagram of the bone-marrow transplant data from ebmt3. Note that this diagram only shows follow-up after PR, as the timescale tfPR is undefined (=missing) for follow-up prior to PR.

## Transitions:

| To |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| From | Tx | PR | RD | RD (PR) | Records: | Events: | Risk time: |  |  |  |  |  |  |  | Persons:

## Rates:

| To |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| From | Tx | PR | RD | RD (PR) | Total |
| Tx | 0 | 0.48 | 0.19 | 0.00 | 0.67 |
| PR | 0 | 0.00 | 0.00 | 0.12 | 0.12 |

In practical analyses this will have no effect, but it is useful in the drawing of multistate models as box-diagrams as shown in the next section. It can also be of interest if simulation-based inference on occupation probabilities for various states are of interest.

## 4 Illustrating the model

In order to illustrate the model in a box diagram, we can use the interactive function boxes.Lexis, which will prompt you for a click for the position of each of the states, and subsequently draw arrows between those states where transitions occur. If we put boxpos=TRUE (the default is FALSE), the states are put in a circular arrangement which usually is not very nice:

```
> boxes( bmtr, boxpos=TRUE )
```



Figure 4: Illustration of the transitions between defined states for the ebmt3 data, using the defualt layout.

That can be improved by using the interactive facility, using the default boxpos=FALSE, which will prompt you to click on the screen to place the various boxes.

Frequently one would need the exact same plot with slight modifications, for example with one or more of the arrows in a different color. To this end it is possible to use the argument file= to specify a file for the code generating the display. This can then be modified:
> boxes( bmtr, file="bmt-boxes.r" )
The following is the content of the file bmt-boxes.r, generated interactively (the annotaion of the code comes from the boxes.Lexis function), and below the resulting plot:

```
obj <- bmtr
if( inherits(obj,"Lexis") ) tm <- tmat.Lexis( obj ) else tm <- obj
### Position of the boxes:
            xx <- c( 16, 75, 24, 82 )
            yy <- c( 71, 78, 21, 26 )
            cex <- 1.5 # How should text and numbers be scaled
        wmult <- 1.5 # Extra box-width relative to string width
        hmult <- 2.25 # Extra box-height relative to string height
        eq.wd <- TRUE # All boxes the same width
        eq.ht <- TRUE # All boxes the same height
        show.Y <- TRUE # Show number of person-years in boxes
    scale.Y <- 1 # How should person-years be scaled
digits.Y <- 1 # How should person-years be printed
    show.D <- TRUE # Show number of events on arrows
    scale.D <- FALSE # How should rates be scaled
digits.D <- 0 # How should rates be printed
                st.nam <- colnames( tm )
if( is.null(st.nam) ) st.nam <- paste(1:ncol(tm))
            pl.nam <- st.nam
            n.st <- length( st.nam )
# If we want to show person-years and events / rates
    SM <- summary(obj,simplify=FALSE,scale=scale.Y)
    Y <- SM[[1]][-n.st-1,"Risk time:"]
    D <- SM[[1+as.logical(scale.D)]][1:n.st,1:n.st] * ifelse(scale.D,scale.D,1)
# No extra line with person-years when they are NA
# and adjust the printing of the person-Years
if( show.Y )
    {
    pl.nam <- gsub( "\\\nNA", "",
                                    paste( st.nam,
                                    formatC( Y, format="f", digits= 1 ,
                                    big.mark=","),
                            sep="\n" ) )
    }
###################################################################
# Here comes the plot
par( mar=c(0,0,0,0), cex=cex )
plot( NA, bty="n",
    xlim=0:1*100, ylim=0:1*100, xaxt="n", yaxt="n", xlab="", ylab="" )
# String height and width only meaningful after a plot has been called
ht <- strheight( pl.nam ) * hmult
wd <- strwidth( pl.nam ) * wmult
# Should all boxes be of same height / width:
if( eq.ht ) ht <- rep( max(ht), length(ht) )
if( eq.wd ) wd <- rep( max(wd), length(wd) )
# Plot the boxes
b <- list()
for( i in 1:n.st )
    b[[i]] <- tbox( pl.nam[i], xx[i], yy[i], wd[i], ht[i] )
# Plot the arrows and the text along them
for( i in 1:n.st ) for( j in 1:n.st )
    {
    if( !is.na(tm[i,j]) )
        {
        arr <- boxarr( b[[i]], b[[j]], offset=!is.na(tm[j,i]) )
        if( show.D ) text( arr$x-arr$d[2], arr$y+arr$d[1],
                formatC( D[i,j], format="f", digits=digits.D ),
```

```
+ adj=as.numeric(c(arr$d[2]>0,arr$d[1]<0)),
+ font=2, col="black" )
+ }
+ }
> # Redraw the boxes with white background to move the arrows behind them
> for( i in 1:n.st ) tbox( pl.nam[i], xx[i], yy[i], wd[i], ht[i], col="white" )
```



Figure 5: Illustration of the transitions between defined states for the ebmt3 data.
The first argument to the boxes function is a Lexis object, and so the defult behaviour is to show the risk time (person-years) in those boxes where they are accumulated, and the number of transitions along the arrows. The first argument may also be a transition matrix, in which case only the boxes and arrows are drawn.

## 5 Analysis of rates

In the first instance we use a Cox-model for the analysis of the three rates. We can either analyse the transitions separately by selecting the appropriate rows and choosing the right event-type. We see that there is minimal difference between the codes; it is only the failure type (from lex.Xst) and the subset (from lex.Cst) that is different:

```
> m.Tx.PR <- coxph( Surv( tft, tft+lex.dur, lex.Xst=="PR" ) ~
+ dissub + age + drmatch + tcd,
+ data = subset(bmtr, lex.Cst=="Tx"), method="breslow" )
> m.Tx.RD <- coxph( Surv( tft, tft+lex.dur, lex.Xst=="RD" ) ~
+ dissub + age + drmatch + tcd,
+ data = subset(bmtr, lex.Cst=="Tx"), method="breslow" )
> m.PR.RD <- coxph( Surv( tft, tft+lex.dur, lex.Xst=="RD(PR)" ) ~
```

```
+ dissub + age + drmatch + tcd,
+ data = subset(bmtr, lex.Cst=="PR"), method="breslow" )
```

We can stack these sets of parameters nicely if we want, using the facilities in ci.lin:

|  | Estimate | StdErr | z | P | $\exp$ (Est.) | 2.5\% | 97.5\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| dissubALL | -0.044 | 0.078 | -0.560 | 0.576 | 0.957 | 0.822 | 1.115 |
| dissubCML | -0.297 | 0.068 | -4.371 | 0.000 | 0.743 | 0.650 | 0.849 |
| age20-40 | -0.165 | 0.079 | -2.082 | 0.037 | 0.848 | 0.726 | 0.990 |
| age>40 | -0.090 | 0.086 | -1.038 | 0.299 | 0.914 | 0.772 | 1.083 |
| drmatchGender mismatch | 0.046 | 0.067 | 0.687 | 0.492 | 1.047 | 0.919 | 1.193 |
| tcdTCD | 0.429 | 0.080 | 5.335 | 0.000 | 1.536 | 1.312 | 1.798 |
| dissubALL | 0.256 | 0.135 | 1.893 | 0.058 | 1.292 | 0.991 | 1.684 |
| dissubCML | 0.017 | 0.108 | 0.155 | 0.877 | 1.017 | 0.822 | 1.258 |
| age20-40 | 0.255 | 0.151 | 1.689 | 0.091 | 1.291 | 0.960 | 1.735 |
| age>40 | 0.526 | 0.158 | 3.334 | 0.001 | 1.693 | 1.242 | 2.307 |
| drmatchGender mismatch | -0.075 | 0.110 | -0.682 | 0.495 | 0.928 | 0.747 | 1.151 |
| tcdTCD | 0.297 | 0.150 | 1.977 | 0.048 | 1.345 | 1.003 | 1.806 |
| dissubALL | 0.137 | 0.148 | 0.923 | 0.356 | 1.146 | 0.858 | 1.532 |
| dissubCML | 0.247 | 0.117 | 2.115 | 0.034 | 1.280 | 1.018 | 1.610 |
| age20-40 | 0.061 | 0.153 | 0.400 | 0.689 | 1.063 | 0.787 | 1.436 |
| age>40 | 0.581 | 0.160 | 3.627 | 0.000 | 1.788 | 1.306 | 2.447 |
| drmatchGender mismatch | 0.173 | 0.115 | 1.510 | 0.131 | 1.189 | 0.950 | 1.488 |
| tcdTCD | 0.201 | 0.126 | 1.589 | 0.112 | 1.222 | 0.954 | 1.566 |

However, in order to analyse the transition rates using joint parameters between transitions, we must do the analyses based on a stacked dataset; that is a dataset which is made by stacking the three subsets we used in the three separate analyses above. Note that these three sets are not disjoint subsets, and moreover the response variable (event indicator) is not the same in all three.

This can conveniently be constructed by using the stack. Lexis function:

```
> bmts <- stack( bmtr )
> str( bmts )
Classes 'stacked.Lexis' and 'data.frame': 5577 obs. of 17 variables:
    $ tft : num 0 0 0 0 0 0 0 0 0 0 ...
    $ tfPR : num NA NA NA NA NA NA NA NA NA NA ...
    $ lex.dur : num 0.063 0.0958 0.0712 0.0602 0.0794 ...
    $ lex.Cst : Factor w/ 4 levels "Tx","PR","RD",..: 1 1 1 1 1 1 1 1 1 1 1 ...
    $ lex.Xst : Factor w/ 4 levels "Tx","PR","RD",..: 2 2 2 2 2 2 2 2 1 3...
    $ lex.Tr : Factor w/ 3 levels "Tx->PR","Tx->RD",..: 1 1 1 1 1 1 1 1 1 1 ...
    $ lex.Fail: logi TRUE TRUE TRUE TRUE TRUE TRUE ...
    $ lex.id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ prtime : num 23 35 26 22 29 38 30 35 NA NA ...
    $ prstat : int 11111111111 0 0 ...
    $ rfstime : num 744 360 135 995 422 ...
    $ rfsstat : int 01101100 0 1 ...
    $ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3 ...
    $ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
    $ drmatch : Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 2 1 1 1 ...
    $ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
```

We see that two extra variables lex.Tr and lex.Fail have been added. The lex.Tr is a factor indicating the transition, and lex.Fail is the (logical) failure indicator for the transition.

Since the variables lex.Cst and lex.Xst have lost their meaning, the resulting object is not a Lexis object.

We can now reproduce the results from above by using the stacked dataset and the interactions with lex. $\operatorname{Tr}$ - note that the interaction with time is given in the strata() argument in the model formula:

```
> c1 <- coxph( Surv(tft,tft+lex.dur,lex.Fail) ~
+ lex.Tr:( dissub + age + drmatch + tcd ) +
+ strata( lex.Tr ),
+ data=bmts, method="breslow" )
> c1
Call:
coxph(formula = Surv(tft, tft + lex.dur, lex.Fail) ~ lex.Tr:(dissub +
    age + drmatch + tcd) + strata(lex.Tr), data = bmts, method = "breslow")
```

|  | coef | exp(coef) | se(coef) | z | p |
| :---: | :---: | :---: | :---: | :---: | :---: |
| lex. $\operatorname{Tr} \mathrm{Tx}$->PR:dissubAML | 0.2972 | 1.346 | 0.0680 | 4.371 | $1.2 \mathrm{e}-05$ |
| lex.TrTx->RD:dissubAML | -0.0167 | 0.983 | 0.1084 | -0.155 | $8.8 \mathrm{e}-01$ |
| lex.TrPR->RD (PR) : dissubAML | -0.2471 | 0.781 | 0.1169 | -2.115 | 3.4e-02 |
| lex.TrTx->PR:dissubALL | 0.2536 | 1.289 | 0.0830 | 3.057 | $2.2 \mathrm{e}-03$ |
| lex. $\operatorname{Tr} \mathrm{Tx}->\mathrm{RD}$ : dissubALL | 0.2391 | 1.270 | 0.1313 | 1.821 | 6.9e-02 |
| lex.TrPR->RD (PR) : dissubALL | -0.1105 | 0.895 | 0.1491 | -0.741 | 4.6e-01 |
| lex.TrTx->PR:dissubCML | NA | NA | 0.0000 | NA | NA |
| lex. $\operatorname{Tr} \mathrm{Tx}->\mathrm{RD}$ : dissubCML | NA | NA | 0.0000 | NA | NA |
| lex.TrPR->RD (PR) : dissubCML | NA | NA | 0.0000 | NA | NA |
| lex.TrTx->PR:age20-40 | -0.1646 | 0.848 | 0.0791 | -2.082 | $3.7 \mathrm{e}-02$ |
| lex.TrTx->RD:age20-40 | 0.2552 | 1.291 | 0.1510 | 1.689 | 9.1e-02 |
| lex.TrPR->RD (PR) : age20-40 | 0.0614 | 1.063 | 0.1534 | 0.400 | $6.9 \mathrm{e}-01$ |
| lex.TrTx->PR:age>40 | -0.0898 | 0.914 | 0.0865 | -1.038 | 3.0e-01 |
| lex.TrTx->RD:age>40 | 0.5265 | 1.693 | 0.1579 | 3.334 | 8.6e-04 |
| lex.TrPR->RD (PR) : age>40 | 0.5809 | 1.788 | 0.1601 | 3.627 | $2.9 \mathrm{e}-04$ |
| lex. $\operatorname{Tr} \mathrm{Tx}->$ PR:drmatchGender mismatch | 0.0458 | 1.047 | 0.0666 | 0.687 | $4.9 \mathrm{e}-01$ |
| lex.TrTx->RD:drmatchGender mismatch | -0.0753 | 0.928 | 0.1103 | -0.682 | 5.0e-01 |
| lex.TrPR->RD (PR):drmatchGender mismatch | 0.1729 | 1.189 | 0.1145 | 1.510 | $1.3 \mathrm{e}-01$ |
| lex.TrTx->PR:tcdTCD | 0.4291 | 1.536 | 0.0804 | 5.335 | 9.6e-08 |
| lex.TrTx->RD:tcdTCD | 0.2967 | 1.345 | 0.1501 | 1.977 | $4.8 \mathrm{e}-02$ |
| lex.TrPR->RD (PR) : tcdTCD | 0.2007 | 1.222 | 0.1264 | 1.589 | $1.1 \mathrm{e}-01$ |

Likelihood ratio test=118 on $18 \mathrm{df}, \mathrm{p}=1.11 \mathrm{e}-16 \mathrm{n}=5577$
We see that the default model matrix set-up for interactions generates an interaction assuming that there is an intercept. Which there indeed is not in a Cox-model. Hence we get aliased parameters when all three levels of dissub have interaction columns with lex. Tr generated.

It gives the correct model, but the reference level for the factor dissub is the last rather than the first. This can be remedied by explicitly tampering with the model matrix before fitting the model; we just generate the model matrix and then remove the columns relating to the reference level. This has the advantage that we maintain the annotation of the parameters:

```
> mm <- model.matrix( ~ lex.Tr:( dissub + age + drmatch + tcd ), bmts )
> rm <- grep( "AML", colnames(mm) )
> mm <- mm[,-c(1,rm)]
> c1 <- coxph(Surv(tft,tft+lex.dur,lex.Fail) ~
+ mm +
+ strata( lex.Tr ),
+ data=cbind(bmts,mm), method="breslow" )
c1
```

Call:
$\operatorname{coxph}(f o r m u l a=\operatorname{Surv}(t f t, t f t+\operatorname{lex} . d u r, \operatorname{lex.Fail}) \sim$ mm + strata(lex.Tr), data $=$ cbind(bmts, mm), method = "breslow")

|  | coef | exp(coef) | se(coef) | z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mmlex.TrTx->PR:dissubALL | -0.0436 | 0.957 | 0.0779 | -0.560 | $5.8 \mathrm{e}-01$ |
| mmlex. $\operatorname{TrTx}->\mathrm{RD}$ : dissubALL | 0.2559 | 1.292 | 0.1352 | 1.893 | $5.8 \mathrm{e}-02$ |
| mmlex.TrPR $->\mathrm{RD}(\mathrm{PR}$ ) : dissubALL | 0.1366 | 1.146 | 0.1480 | 0.923 | $3.6 \mathrm{e}-01$ |
| mmlex.TrTx->PR:dissubCML | -0.2972 | 0.743 | 0.0680 | -4.371 | $1.2 \mathrm{e}-05$ |
| mmlex.TrTx->RD:dissubCML | 0.0167 | 1.017 | 0.1084 | 0.155 | 8.8e-01 |
| mmlex.TrPR $->\mathrm{RD}(\mathrm{PR}$ ) : dissubCML | 0.2471 | 1.280 | 0.1169 | 2.115 | $3.4 \mathrm{e}-02$ |
| mmlex.TrTx->PR:age20-40 | -0.1646 | 0.848 | 0.0791 | -2.082 | $3.7 \mathrm{e}-02$ |
| mmlex.TrTx->RD:age20-40 | 0.2552 | 1.291 | 0.1510 | 1.689 | 9.1e-02 |
| mmlex.TrPR->RD (PR) : age20-40 | 0.0614 | 1.063 | 0.1534 | 0.400 | $6.9 \mathrm{e}-01$ |
| mmlex.TrTx->PR:age>40 | -0.0898 | 0.914 | 0.0865 | -1.038 | $3.0 \mathrm{e}-01$ |
| mmlex.TrTx $->\mathrm{RD}$ : age $>40$ | 0.5265 | 1.693 | 0.1579 | 3.334 | 8.6e-04 |
| mmlex.TrPR->RD (PR) : age>40 | 0.5809 | 1.788 | 0.1601 | 3.627 | $2.9 \mathrm{e}-04$ |
| mmlex.TrTx->PR:drmatchGender mismatch | 0.0458 | 1.047 | 0.0666 | 0.687 | $4.9 \mathrm{e}-01$ |
| mmlex.TrTx->RD:drmatchGender mismatch | -0.0753 | 0.928 | 0.1103 | -0.682 | $5.0 \mathrm{e}-01$ |
| mmlex.TrPR->RD(PR) : drmatchGender mismatch | 0.1729 | 1.189 | 0.1145 | 1.510 | $1.3 \mathrm{e}-01$ |
| mmlex.TrTx->PR:tcdTCD | 0.4291 | 1.536 | 0.0804 | 5.335 | 9.6e-08 |
| mmlex.TrTx->RD:tcdTCD | 0.2967 | 1.345 | 0.1501 | 1.977 | $4.8 \mathrm{e}-02$ |
| mmlex.TrPR->RD(PR) : tcdTCD | 0.2007 | 1.222 | 0.1264 | 1.589 | $1.1 \mathrm{e}-01$ |

Likelihood ratio test=118 on $18 \mathrm{df}, \mathrm{p}=1.11 \mathrm{e}-16 \mathrm{n}=5577$
Comparison with the estimates from the 3 separate analyses and from the mstate paper we can just use ci.lin to extract estimates, and the reshuffle the rows:

```
> round( ci.lin( c1, E=T )[c(0:5*3+1,0:5*3+2,0:5*3+3),-(3:4)], 3 )
```

mmlex.TrTx->PR:dissubALL
mmlex.TrTx->PR:dissubCML
mmlex.TrTx->PR:age20-40
mmlex.TrTx->PR:age>40
mmlex.TrTx->PR:drmatchGender mismatch
mmlex.TrTx->PR:tcdTCD
mmlex.TrTx->RD:dissubALL
mmlex.TrTx->RD:dissubCML
mmlex.TrTx->RD:age20-40
mmlex.TrTx->RD: age>40
mmlex.TrTx->RD:drmatchGender mismatch
mmlex.TrTx->RD:tcdTCD
mmlex.TrPR->RD(PR):dissubALL
mmlex.TrPR->RD(PR): dissubCML
mmlex.TrPR->RD(PR): age20-40

| Estimate | StdErr | $\exp$ (Est.) | $2.5 \%$ | $97.5 \%$ |
| ---: | ---: | ---: | ---: | ---: |
| -0.044 | 0.078 | 0.957 | 0.822 | 1.115 |
| -0.297 | 0.068 | 0.743 | 0.650 | 0.849 |
| -0.165 | 0.079 | 0.848 | 0.726 | 0.990 |
| -0.090 | 0.086 | 0.914 | 0.772 | 1.083 |
| 0.046 | 0.067 | 1.047 | 0.919 | 1.193 |
| 0.429 | 0.080 | 1.536 | 1.312 | 1.798 |
| 0.256 | 0.135 | 1.292 | 0.991 | 1.684 |
| 0.017 | 0.108 | 1.017 | 0.822 | 1.258 |
| 0.255 | 0.151 | 1.291 | 0.960 | 1.735 |
| 0.526 | 0.158 | 1.693 | 1.242 | 2.307 |
| -0.075 | 0.110 | 0.928 | 0.747 | 1.151 |
| 0.297 | 0.150 | 1.345 | 1.003 | 1.806 |
| 0.137 | 0.148 | 1.146 | 0.858 | 1.532 |
| 0.247 | 0.117 | 1.280 | 1.018 | 1.610 |
| 0.061 | 0.153 | 1.063 | 0.787 | 1.436 |

```
mmlex.TrPR->RD(PR):age>40 llllllll
mmlex.TrPR->RD(PR):drmatchGender mismatch }\quad0.173 0.115 1.189 0.950 1.488
mmlex.TrPR->RD(PR):tcdTCD }\quad0.201 0.126 1.222 0.954 1.566
```


### 5.1 Joint parameters between transitions

Of course it is not of much interest to make a joint model which gives the same results as the three separate ones. The point comes from reducing this model to one which is more parsimonious yet sensible. One such reduction would be to assume that the two relapse/mortality rates, $\mathrm{Tx}->\mathrm{RD}$ and $\mathrm{PR}->\mathrm{RD}$ are proportional. This means that the Cox model should be changed so that the stratum variable only has two levels, but at the same time we should introduce an indicator of being in state PR. We use the Relevel command from Epi that allows grouping of factor levels, and repeat the tampering with the model matrix:

```
> bmts$inc.mort <- Relevel( bmts$lex.Tr, list(1,2:3), coll=" & " )
> with( bmts, table(lex.Tr,inc.mort) )
\begin{tabular}{|c|c|c|}
\hline & inc.mort & \\
\hline lex.Tr & Tx->PR Tx->RD & \& PR->RD (PR) \\
\hline Tx \(->P \mathrm{R}\) & 2204 & 0 \\
\hline Tx \(->\mathrm{RD}\) & 0 & 2204 \\
\hline PR \(->\mathrm{RD}\) (PR) & 0 & 1169 \\
\hline
\end{tabular}
> mm <- model.matrix( ~ lex.Tr:( dissub + age + drmatch + tcd ), bmts )
> rm <- grep( "AML", colnames(mm) )
> mm <- mm[,-c(1,rm)]
> c2 <- coxph( Surv(tft,tft+lex.dur,lex.Fail) ~
+ mm + I(lex.Cst=="PR") +
+ strata( inc.mort ),
+ data=cbind(bmts,mm), method="breslow" )
```

Note that the logical expression identifying platelet recovery is wrapped in an "I ()" in order to make coxph create an extra column adjacent to the supplied model matrix (if not coxph will crash):

```
> c2
Call:
coxph(formula = Surv(tft, tft + lex.dur, lex.Fail) ~ mm + I(lex.Cst ==
    "PR") + strata(inc.mort), data = cbind(bmts, mm), method = "breslow")
```

```
mmlex.TrTx->PR:dissubALL
mmlex.TrTx}->\mathrm{ RD:dissubALL
mmlex.TrPR->RD(PR):dissubALL
mmlex.TrTx->PR:dissubCML
mmlex.TrTx->RD:dissubCML
mmlex.TrPR->RD(PR):dissubCML
mmlex.TrTx->PR:age20-40
mmlex.TrTx->RD:age20-40
mmlex.TrPR->RD(PR): age20-40
mmlex.TrTx->PR:age>40
```

| coef | exp(coef) | se(coef) | z | p |
| ---: | ---: | ---: | ---: | ---: |
| -0.04359 | 0.957 | 0.0779 | -0.5597 | $5.8 \mathrm{e}-01$ |
| 0.26100 | 1.298 | 0.1352 | 1.9308 | $5.4 \mathrm{e}-02$ |
| 0.13993 | 1.150 | 0.1480 | 0.9456 | $3.4 \mathrm{e}-01$ |
| -0.29724 | 0.743 | 0.0680 | -4.3714 | $1.2 \mathrm{e}-05$ |
| 0.00351 | 1.004 | 0.1084 | 0.0324 | $9.7 \mathrm{e}-01$ |
| 0.25051 | 1.285 | 0.1168 | 2.1450 | $3.2 \mathrm{e}-02$ |
| -0.16461 | 0.848 | 0.0791 | -2.0823 | $3.7 \mathrm{e}-02$ |
| 0.25083 | 1.285 | 0.1511 | 1.6605 | $9.7 \mathrm{e}-02$ |
| 0.05554 | 1.057 | 0.1534 | 0.3621 | $7.2 \mathrm{e}-01$ |
| -0.08979 | 0.914 | 0.0865 | -1.0384 | $3.0 \mathrm{e}-01$ |


| mmlex.TrTx->RD:age>40 | 0.52577 | 1.692 | 0.1579 | 3.3299 | $8.7 \mathrm{e}-04$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| mmlex.TrPR->RD(PR) : age>40 | 0.56267 | 1.755 | 0.1600 | 3.5173 | $4.4 \mathrm{e}-04$ |
| mmlex.TrTx->PR:drmatchGender mismatch | 0.04575 | 1.047 | 0.0666 | 0.6869 | $4.9 \mathrm{e}-01$ |
| mmlex.TrTx->RD:drmatchGender mismatch | -0.07205 | 0.930 | 0.1103 | -0.6535 | $5.1 \mathrm{e}-01$ |
| mmlex.TrPR->RD(PR) :drmatchGender mismatch | 0.16939 | 1.185 | 0.1144 | 1.4801 | $1.4 \mathrm{e}-01$ |
| mmlex.TrTx->PR:tcdTCD | 0.42907 | 1.536 | 0.0804 | 5.3346 | $9.6 \mathrm{e}-08$ |
| mmlex.TrTx->RD:tcdTCD | 0.31886 | 1.376 | 0.1500 | 2.1262 | $3.3 \mathrm{e}-02$ |
| mmlex.TrPR->RD(PR) :tcdTCD | 0.21103 | 1.235 | 0.1262 | 1.6722 | $9.4 \mathrm{e}-02$ |
| I(lex.Cst == "PR")TRUE | -0.38036 | 0.684 | 0.2115 | -1.7982 | $7.2 \mathrm{e}-02$ |

Likelihood ratio test=136 on $19 \mathrm{df}, \mathrm{p}=0 \mathrm{n}=5577$

## 6 Connecting to the mstate package

Alternatively we may throw away some of the state annotation and convenience in parameter labeling and use the mstate package:

```
> ms <- mstate( bmt )
> head( bmt )
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & & lex.dur & lex.Cst & x. & lex.id & d & pr & prst & rfstime & rf & dis & age & & \\
\hline 1 & 0 & 2.0369610 & Tx & Tx & 1 & 1 & 23 & 1 & 744 & 0 & CML & >40 & & Gen \\
\hline 2 & 0 & 0.9856263 & Tx & RD & 2 & 2 & 35 & 1 & 360 & 1 & CML & >40 & No & gen \\
\hline 3 & 0 & 0.3696099 & Tx & RD & 3 & 3 & 26 & 1 & 135 & 1 & CML & >40 & No & ge \\
\hline 4 & 0 & 2.7241615 & Tx & Tx & 4 & 4 & 22 & 1 & 995 & 0 & AML & 20-40 & No & gen \\
\hline 5 & 0 & 1.1553730 & Tx & RD & 5 & 5 & 29 & 1 & 422 & 1 & AML & 20-40 & No & \\
\hline 6 & 0 & 0.3258042 & Tx & RD & 6 & 6 & 38 & 1 & 119 & 1 & ALL & >40 & No & gen \\
\hline
\end{tabular}
```

> head( ms )

|  | id | Tstart | Tstop | from | to trans | status | id. 1 | prtime | prstat | rfstime | rfsstat dissub | age |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 0 | 2.0369610 | 1 | 1 | 1 | FALSE | 1 | 23 | 1 | 744 | 0 | CML |
| 2 | 2 | 0 | 0.9856263 | 1 | 1 | 1 | TRUE | 2 | 35 | 1 | 360 | 1 | CML |
| 3 | 3 | 0 | 0.3696099 | 1 | 1 | 1 | TRUE | 3 | 26 | 1 | 135 | 1 | CML |
| 4 | 4 | 0 | 2.7241615 | 1 | 1 | 1 | FALSE | 4 | 22 | 1 | 995 | 0 | AML |
| 4 | $20-40$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 5 | 0 | 1.1553730 | 1 | 1 | 1 | TRUE | 5 | 29 | 1 | 422 | 1 | AML |
| 6 | 6 | 0 | 0.3258042 | 1 | 1 | 1 | TRUE | 6 | 38 | 1 | 119 | 1 | ALL |$>40$

## 7 Poisson modeling

An alternative to the Cox-modeling of the rates is to use a Poisson model; i.e. to assume that rates are constant in (small) intervals, and then invoke a model for the rates in these intervals that assume that the rates are constant in each interval, but that the magnitude of the rates follow some smooth function.

In order for this to make sense we must split the data along one or more of the timescales of relevance. We split along the timescale tft (time from transplant). This is accomplished by the function splitLexis (note that it is not called split.Lexis):

```
> bmtx <- splitLexis( bmtr, time.scale="tft", breaks=c(0:19/10,seq(2,10,0.5)) )
> summary( bmtx )
```



We see that the number of events and amount of risk time is unchanged in bmtx; the only difference is that it is distributed across many more records than in bmtr:

```
> summary( bmtr )
Transitions:
        To
\begin{tabular}{crrrrrrr} 
From & Tx & PR & RD & RD(PR) & Records: & Events: & Risk time:
\end{tabular} Persons:
Rates:
            To
From Tx PR RD RD(PR) Total
    Tx 0 0.48 0.19 0.00 0.67
    PR 0 0.00 0.00 0.12 0.12
```

We can now redo the rate-modeling above by using a Poisson model with a suitable smooth version of the time-effect.

```
> library( splines )
> p.Tx.PR <- glm( as.numeric( lex.Xst=="PR" ) ~
+ ns(tft, knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) ) +
+ dissub + age + drmatch + tcd,
+ family=poisson,
+ data = subset(bmtx, lex.Cst=="Tx") )
> p.Tx.RD <- glm( as.numeric( lex.Xst=="RD" ) ~
+ ns( tft, knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) ) +
+ dissub + age + drmatch + tcd,
+ family=poisson,
+ data = subset(bmtx, lex.Cst=="Tx") )
> p.PR.RD <- glm( as.numeric( lex.Xst=="RD(PR)" ) ~
+ ns( tft, knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) ) +
+ dissub + age + drmatch + tcd,
+ family=poisson,
+ data = subset(bmtx, lex.Cst=="PR") )
```

We then compare the results for the regression parameters from the Poisson-model with those from the Cox-model:

```
> sb <- c("dissub","age","drmatch","tcd")
> round( rbind( ci.lin( p.Tx.PR, subset=sb, E=T ),
+ ci.lin( p.Tx.RD, subset=sb, E=T ),
+ ci.lin( p.PR.RD, subset=sb, E=T ) ), 3 )
```



It gives a better impression if we divide the two sets of regression estimates to see the relative differences in RR estimates (the last 3 columns):

```
> round( rbind( ci.lin( m.Tx.PR, E=T ),
+ ci.lin(m.Tx.RD, E=T ),
+ ci.lin(m.PR.RD, E=T ) ) /
+ rbind( ci.lin( p.Tx.PR, subset=sb, E=T ),
+ ci.lin( p.Tx.RD, subset=sb, E=T ),
+ ci.lin( p.PR.RD, subset=sb, E=T ) ), 3 )
```

dissubALL
dissubCML
age20-40
age>40
drmatchGender mismatch
tcdTCD
dissubALL
dissubCML
age20-40
age>40
drmatchGender mismatch
tcdTCD
dissubALL
dissubCML
age20-40
age>40
drmatchGender mismatch
tcdTCD

| Estimate | StdErr | $z$ | P | $\exp$ (Est.) | $2.5 \%$ | $97.5 \%$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 3.386 | 0.998 | 3.392 | 0.663 | 0.970 | 0.970 | 0.970 |
| 1.229 | 1.000 | 1.229 | 0.033 | 0.946 | 0.946 | 0.946 |
| 1.042 | 1.001 | 1.041 | 0.822 | 0.993 | 0.993 | 0.994 |
| 0.815 | 0.996 | 0.818 | 1.463 | 1.021 | 1.021 | 1.020 |
| 0.777 | 1.000 | 0.777 | 1.307 | 0.987 | 0.987 | 0.987 |
| 1.089 | 1.003 | 1.086 | 0.107 | 1.036 | 1.035 | 1.036 |
| 1.002 | 1.000 | 1.002 | 0.991 | 1.000 | 1.001 | 1.000 |
| 0.469 | 1.000 | 0.469 | 1.183 | 0.981 | 0.981 | 0.981 |
| 0.956 | 1.000 | 0.956 | 1.179 | 0.988 | 0.988 | 0.989 |
| 0.997 | 1.000 | 0.997 | 1.042 | 0.998 | 0.998 | 0.998 |
| 1.041 | 1.000 | 1.041 | 0.966 | 0.997 | 0.997 | 0.997 |
| 1.158 | 1.000 | 1.158 | 0.547 | 1.041 | 1.041 | 1.041 |
| 1.034 | 1.001 | 1.033 | 0.958 | 1.005 | 1.004 | 1.005 |
| 0.965 | 1.000 | 0.965 | 1.216 | 0.991 | 0.991 | 0.991 |
| 0.933 | 1.001 | 0.932 | 1.032 | 0.996 | 0.995 | 0.996 |
| 1.005 | 1.001 | 1.004 | 0.944 | 1.003 | 1.003 | 1.003 |
| 1.052 | 1.000 | 1.052 | 0.868 | 1.009 | 1.009 | 1.009 |
| 0.974 | 1.000 | 0.974 | 1.089 | 0.995 | 0.995 | 0.995 |

As before we can model this in one go by stacking the dataset using stack.Lexis. Note that it is impossible to try and use splitLexis on a stacked dataset because it is no longer a Lexis object. Hence stacking must be done after splitting data.

```
> bmtxs <- stack( bmtx )
> str( bmts )
```

```
Classes 'stacked.Lexis' and 'data.frame': 5577 obs. of 18 variables:
    $ tft : num 0 0 0 0 0 0 0 0 0 0 ...
    $ tfPR : num NA NA NA NA NA NA NA NA NA NA ...
    $ lex.dur : num 0.063 0.0958 0.0712 0.0602 0.0794 ...
    $ lex.Cst : Factor w/ 4 levels "Tx","PR","RD",..: 1 1 1 1 1 1 1 1 1 1 ...
    $ lex.Xst : Factor w/ 4 levels "Tx","PR","RD",..: 2 2 2 2 2 2 2 2 1 3 ...
    $ lex.Tr : Factor w/ 3 levels "Tx->PR","Tx->RD",..: 1 1 1 1 1 1 1 1 1 1 %..
    $ lex.Fail: logi TRUE TRUE TRUE TRUE TRUE TRUE ...
    $ lex.id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ id : int 1 2 3 4 5 6 7 8 9 10 ...
    $ prtime : num 23 35 26 22 29 38 30 35 NA NA ...
    $ prstat : int 1111111111 0 0 ...
    $ rfstime : num 744 360 135 995 422 ...
    $ rfsstat : int 0 1 1 0 1 1 0 0 0 1 ...
    $ dissub : Factor w/ 3 levels "AML","ALL","CML": 3 3 3 1 1 2 3 2 1 3...
    $ age : Factor w/ 3 levels "<=20","20-40",..: 3 3 3 2 2 3 2 1 2 3 ...
    $ drmatch : Factor w/ 2 levels "No gender mismatch",..: 2 1 1 1 1 1 2 1 1 1 ...
    $ tcd : Factor w/ 2 levels "No TCD","TCD": 1 1 1 1 1 1 1 1 1 1 ...
    $ inc.mort: Factor w/ 2 levels "Tx->PR","Tx->RD & PR->RD(PR)": 1 1 1 1 1 1 1 1 1 1 1 ...
```

We can now redo the analysis with in one step. Moreover we can now get a global test for the proportionality of the two mortality rates as a simple likelihood-ratio test. Note that in order to test for proportionality of rates between the two mortality rates, we must have an interaction between the timescale (ns (tft,...) and a factor where the two mortalities are grouped together. But we must still have a main effect of lex. Tr , the type of transition:

```
> p1 <- glm( as.numeric(lex.Fail) ~
+ lex.Tr +
+ lex.Tr:( ns(tft, knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) ) +
    dissub + age + drmatch + tcd ),
+ family=poisson,
+ data=bmtxs )
```

To fit a model with proportional mortality rates (from Tx and from PR) we must group the two transitions into one level, but in order to avaoid a model that assumes identical mortalities, we still include the three level factor lex. $\operatorname{Tr}$ as a main effect:

```
> bmtxs$inc.mort <- Relevel( bmtxs$lex.Tr, list(1,2:3), coll=" & " )
> p2 <- glm( as.numeric(lex.Fail) ~
+ lex.Tr +
+ inc.mort:ns(tft, knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) ) +
+ lex.Tr:( dissub + age + drmatch + tcd ),
+ family=poisson,
+ data=bmtxs )
> round( ci.lin( p2, subset=sb, E=T ), 3 ) [,5:7]
\begin{tabular}{lrrl} 
lex.TrTx->PR:dissubALL & 0.987 & 0.847 & 1.150 \\
lex.TrTx->RD:dissubALL & 1.297 & 0.995 & 1.691 \\
lex.TrPR->RD (PR):dissubALL & 1.142 & 0.855 & 1.526 \\
lex.TrTx->PR:dissubCML & 0.785 & 0.687 & 0.897 \\
lex.TrTx->RD:dissubCML & 1.021 & 0.826 & 1.263 \\
lex.TrPR->RD (PR):dissubCML & 1.295 & 1.030 & 1.628 \\
lex.TrTx->PR:age20-40 & 0.854 & 0.731 & 0.997 \\
lex.TrTx->RD:age20-40 & 1.297 & 0.965 & 1.744 \\
lex.TrPR->RD (PR):age20-40 & 1.059 & 0.785 & 1.430
\end{tabular}
```

```
lex.TrTx->PR:age>40
0.896 0.756 1.062
lex.TrTx->RD:age>40 1.695 1.243 2.309
lex.TrPR->RD(PR): age>40
1.741 1.273 2.382
lex.TrTx}->PR:drmatchGender mismatch
lex.TrTx->RD:drmatchGender mismatch
1.061 0.931 1.209
0.932 0.751 1.157
lex.TrPR->RD(PR):drmatchGender mismatch
1.174 0.938 1.469
lex.TrPR >RD(PR):drmatchGender mismatch
lex.TrTx->PR:tcdTCD
1.483 1.267 1.735
lex.TrTx->RD:tcdTCD
lex.TrPR->RD(PR):tcdTCD
1.332 0.993 1.786
1.243 0.971 1.592
> anova( p1, p2, test="Chisq" )
Analysis of Deviance Table
Model 1: as.numeric(lex.Fail) ~ lex.Tr + lex.Tr:(ns(tft, knots = c(seq(0.2,
        2, 0.2), 3:6), Bo = c(0, 7)) + dissub + age + drmatch + tcd)
Model 2: as.numeric(lex.Fail) ~ lex.Tr + inc.mort:ns(tft, knots = c(seq(0.2,
        2, 0.2), 3:6), Bo = c(0, 7)) + lex.Tr:(dissub + age + drmatch +
        tcd)
    Resid. Df Resid. Dev Df Deviance P(>|Chi|)
1 55266 9125.4
2 
```

The test is highly significant, but it would be relevant to see what the actual differences in shape between the two mortality rates were.

Hence we extract the effect of tft from the model p 1 for the transitions $\mathrm{Tx} \rightarrow \mathrm{RD}$ and $\mathrm{PR} \rightarrow \mathrm{RD}$.

To this end we construct a contrast matrix to multiply with the relevant spline parameters, by setting up the splines for a predefined set of times. Since we are mainly interested in the relative behaviour of the rates, we compute the RR relative to the mortality 1 year after entry:

```
> p.pt <- seq(0,8,,200)
> CM <- ns( p.pt , knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) )
>C1 <- ns( rep(1,length(p.pt)), knots=c(seq(0.2,2,0.2),3:6), Bo=c(0,7) )
```

This matrix can now be applied to the estimates for the splines for the two transitions. The relevant parameters are selected using the subset argument to ci.lin. The Exp=TRUE gives the exponential of the computed parameter functions with c.i. i columns 5 to 7 of the result:

```
> rr.Tx <- ci.lin( p1, subset="Tx->RD:ns", ctr.mat=CM-C1, Exp=TRUE ) [,5:7]
> rr.PR <- ci.lin( p1, subset="PR->RD\\(PR\\):ns", ctr.mat=CM-C1, Exp=TRUE ) [,5:7]
```

Note the use of "<br>" in the subset argument - it is because "(" and ")" have special meanings in regular expressions, so the characters need to be escaped in order to work properly. However, we would like the see the estimated rate-ratio between the two transitions as well. This is done by extracting the two sets of parameters and them multiplying them by a contrast matrix which is made by joining the contrast matrix by the negative of the same:

```
> RR <- ci.lin( p1, subset=c("Tx->RD:ns","PR->RD\\(PR\\):ns"),
+ ctr.mat=cbind(CM,-CM), Exp=TRUE )[,5:7]
```

```
> matplot( p.pt, cbind( rr.Tx, rr.PR ),
+ log="y", ylim=c(0.1,10)/2,
+ type="l", lty=1, lwd=c(3,1,1), col=rep(c("red","blue"),each=3) )
> points( 1, 1, pch=16 )
> matlines( p.pt, RR, type="l", lty=1, lwd=c(3,1,1), col="black" )
```

Clearly the underlying rates are over-modeled (although not to the extent as in the Cox-model), but also it is pretty clear that despite the overmodeling there is a tendency that the mortality rates for those in platelet recovery (PR, blue line) do not decrease so rapidly the first two years. Apparently the entire difference takes place during the first year.


Figure 6: $R R$ relative to year 1 after transplant for persons in states Tx (red) and PR (blue), as well as the RR between the mortality rates in Tx and PR (black). Thin lines are estimated 95\% confidence intervals.

## 8 Competing risks

The special case of a multistate model with only one initial state ("alive") and a number of absorbing states ("causes of death"), we can get away with setting up a Lexis object particularly simple. There are two defaults that are used in the setup of the object for the aidssi data:

1. If only the entry argument is omitted, all entries are assumed to be at 0 at the only timescale - it is non-sensical to have more timescales in this instance.
2. If no entry.state is specified is by default assumed that all entires are in the state which is the first level of the exit.state.

Thus we must make sure that the "event-free" is the first level of the status factor:

```
> data(aidssi)
> LA <- Lexis( exit = list(zeit=time),
+ exit.status = factor(cause,levels=c("event-free","AIDS","SI")),
+ data = aidssi )
NOTE: entry.status has been set to "event-free" for all.
NOTE: entry is assumed to be O on the zeit timescale.
> summary( LA )
Transitions:
        To
\begin{tabular}{crrrrrr} 
From & event-free & AIDS & SI & Records: & Events: Risk time: & Persons: \\
event-free & 107 & 114 & 108 & 329 & 222 & 2274.55
\end{tabular}
Rates:
            To
From event-free AIDS SI Total
    event-free 00.05 0.05 0.1
```

Figure 7 is the result of using the default layout of boxes.Lexis:

```
> boxes( LA, boxpos=TRUE )
```



Figure 7: Illustration of the transitions in the competing risks model for the aidssi data.

